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Theme: Culture meets culture, mathematics in and around us

25 – 29 June 2018

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FOREWORD

Welcome to the Free State and to the 24th Annual National Congress of the Association for Mathematics Education of South Africa.

The Congress theme for 2018 "*Culture meets culture, mathematics in and around us"* is a very appropriate one in our uniquely South African context. South Africa is a diverse land of many cultures and languages. Many people have contributed to South Africa being a successful and thriving country, despite some of the challenges we face. AMESA plays an important role in unifying our people through our commitment to quality teaching and learning in our schools.

Mathematics has always been an important subject in South Africa as well as other countries. When learners experience success in mathematics it is likely to result in them choosing careers in the sciences, health sciences, engineering, commerce or other key professions.

For learners to achieve success in mathematics, it is vital that our mathematics teachers are qualified in the subject and are able to deliver quality lessons to learners. These quality lessons will make learning more meaningful, relevant and appropriate. In seeing mathematics as part of our everyday life and culture, and by making it more accessible, we hope to restore the joy of doing mathematics across all grades, and in this way improve learner performance.

For far too long in South Africa, Mathematics as a school subject has been a sifter rather than an enabler. It sifted learners out of the scarce careers that our country so desperately needs. The introduction of Mathematical Literacy has, unwittingly, aggravated this problem. Mathematical Literacy was introduced to give learners, who would not normally take up Mathematics in Grade 10, the opportunity of leaving Grade 12 with some basic or elementary mathematical knowledge to enable them to function effectively as a citizen in the 21st century. Unfortunately, the focus in reality is for schools to improve their overall pass rates, rather than having quality passes.

The 2018 Congress theme calls on all AMESA members and mathematics teachers to refocus their efforts in classrooms on quality teaching and learning. Mathematics must be alive and part of our everyday lives and educational culture.

A large number of long and short papers were submitted for the 2018 AMESA Congress. The papers cover topics ranging from the implications of the NBTP Quantitative Literacy test results for teachers, to the effective application of instructional leadership as a strategy to lead mathematics teachers.

The papers in the two volumes are grouped as follows: Volume 1 consists of plenary and long papers; Volume 2 consists of short papers, workshops and "how I teach papers". In addition, a CD rom comprising both Volume 1 and Volume 2 papers and other activities, has also been produced.

All long and short papers have been reviewed by at least three reviewers. Some papers required minor revisions and others more major revisions. In both cases, authors had to revise their papers accordingly and tabulate these changes so that the academic committee could make the necessary decision on the papers.

Our thanks go to Vasuthavan Govender for his assistance in the preparation of the proceedings.

Karen Junqueira and Rajendran Govender

AMESA editorial team, 2018

ACKNOWLEDGEMENTS

All papers submitted to the congress were sent for triple-blind reviewing.

Many thanks to our review team who reviewed the papers in a constructive way.

Names:

Eben Swanepoel
Jannie Pretorius
Karen Junqueira
Lukanda Kalobo
Lynette Jacobs
Rajendran Govender
Tshele Moloi
Vasuthavan Govender

REVIEW PROCESS

Each of the short paper submissions accepted for publication in this volume of the Proceedings were subject to blind peer review by experienced mathematics educators. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submissions.

Number of short paper submissions:	10
Number of 1-hour workshop submissions:	8
Number of 2-hour workshop submissions:	16
Number of How I Teach paper submissions:	7
Number of submissions accepted:	37
Number of submissions rejected:	4
Number of submissions withdrawn by authors:	0

We thank the reviewers for giving their time and expertise to reviewing the submissions.

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EXPLORING MATHEMATICS TEACHERS' USE OF TECHNOLOGY-BASED TOOLS TO TEACH GRADE 10 EUCLIDEAN GEOMETRY IN FOUR SELECTED SCHOOLS IN KWAZULU-NATAL, SOUTH AFRICA

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The current study seeks to answer the following critical questions: 1. What technology-based tools do Mathematics teachers use when teaching Euclidean Geometry in Grade 10 in KwaZulu-Natal? 2. How and why do the technology-based tools that Mathematics teachers use, promote or constrain the learning of Euclidean Geometry in Grade 10 in KwaZulu-Natal? The case study will be conducted in KwaZulu-Natal, South Africa. It is embedded within the Activity Theory and it will be conducted within the qualitative interpretive framework. Four secondary schools which use technology-based tools to teach Euclidean geometry were chosen purposively. One teacher from each school will be chosen unless they use different technology-based tools in which case two teachers will be chosen. Three Grade 10 Euclidean geometry lessons will be observed from each school. Each lesson will be video recorded. Teacher interviews will be conducted and their responses will be audio recorded. Then two learner focus groups from each school, comprising six learners each, will be interviewed and their responses will be audio recorded.

INTRODUCTION

In the current study, by technology I am referring to any equipment or tools which are hardware devices such as data projectors, white boards and software programs such as the Geometer"s Sketchpad, Geogebra, Cabri, Cinderella, WhatsApp, twitter, internet, Google apps for education (GAFE) and others

Focus

The current study focuses on the use of technology-based tools by Mathematics teachers to teach Grade 10 Euclidean geometry in four selected schools in KwaZulu-Natal, South Africa. It seeks to answer the following critical questions: 1. What technology-based tools do Mathematics teachers use when teaching Euclidean Geometry in Grade 10 in KwaZulu-Natal? 2. How and why do the technology-based tools that Mathematics teachers use, promote or constrain the learning of Euclidean Geometry in Grade 10 in KwaZulu-Natal?

Motivation

It is motivated by companies such as MTN, Vodacom, Cell C who have donated technology-based tools like tablets, white boards, data projectors, computers, lap tops and e-books to secondary schools. Their focus is in the learning of Mathematics in Grade 10 as they aim to enhance understanding of Mathematics in Grade 10 a first year class in the FET phase. Grade 10 is the class in South Africa where learners choose between Mathematics and Mathematical Literacy. The current study is also inspired by United, Educational, Scientific and Cultural Organization who organized an international symposium and Policy Forum based on the theme: Cracking the Code: Girls‟ education in STEM, in Bangkok Thailand on 28-30 August 2017. The symposium focused on the fact that very few girls and women study Science, Technology, Engineering and Mathematics (STEM) subjects.

THEORETICAL FRAMEWORK

The current study is embedded within the Activity Theory which looks at who is doing what, how and why. Video recording of the lessons that will be observed, therefore, best fits this theoretical framework. Often what people seem to be doing, what they say they are doing, and what they actually do, can be quite different (Hassan & Kazlauskas, 2014). Activity Theory is grounded in the work of Vygotsky and his students in the 1920s. The current study seeks to study the complexities of real world situations, modern workplaces which are places of learning, secondary schools. Activity Theory provides a language and a set of frameworks for making sense of what is discovered about the situation through observation, interviews and other methods. Using the Activity Theory lens for the current research, takes activity as the unit of analysis, where activity is defined by the dialectic relationship between subject and object, in other words, „who is doing what for what purpose‟ (Vygotsky,1978).

The current study requires gaining an in-depth knowledge and greater understanding of methods used in the teaching of Mathematics in secondary schools. It will thus be conducted within the qualitative interpretive framework. A qualitative case study for the current study will provide the deeper meaning of how Mathematics teachers use technology-based tools to teach Euclidean Geometry in Grade 10. Baxter and Jack (2008) described a qualitative case study as an approach to research that facilitates and explores a phenomenon within its context using a variety of data sources. Furthermore, this ensures that the issue is explored through a variety of lenses which allows for multiple facets of the phenomenon to be understood.

Activity Theory Model contextualizes interaction between humans and computers (Attwell & Hughes (2010). It has a doer (subject) and an object (the deed) which form an outcome. The relationship between the subject and the object, form the core of an activity. The outcomes of an activity can be the intended ones and some the unintended. The subject-object relationship at the core of an activity is referred to as dialectic.

LITERATURE

There are various apps and software which can be used to learn Mathematics. The internet, the Geometer‟s Sketchpad, Geogebra and Cabri 3D. Technological, pedagogical, content knowledge will be briefly discussed.

Brown and Adler (2008) asserted that the most intense impact of the Internet is its ability to support and expand the various aspects of social learning. The Geometer‟s Sketchpad is interactive geometry software that is used to explore Algebra, Euclidean geometry, Calculus and other areas of Mathematics. It can be used from grade 3 through to university. It has built-in facilities which enable users to construct, measure, transform and animate objects. Geogebra is software which allows one to work with dynamic and differential equations, symbolic algebra, graphing, probability, dynamic 2D and 3D geometry, dynamic texts, spreadsheets and fitting functions of any kind of data (Jonas & Lingefjard, 2017). It works on computers, iPhones under multiple operating systems and on tablet computers and it can be used over sixty-five different languages. Cabri 3D is a software which connects algebra and geometry such that length, distance, area, angles, volume can be measured and be used in algebraic expressions. Cabri 3D also gives opportunities for teaching 3D Euclidean geometry.

Teachers, who want to integrate technology into their teaching, need to be competent in three domains, content, pedagogy and the potential of technology (Voogt, Visser, Roblin, Tonder & van Braak, 2012). Voogt *et al* (2012) defined technological, pedagogical, content knowledge (TPACK) as a conceptual framework for the knowledge base teachers need to effectively teach with technology. Furthermore, teacher TPACK and beliefs about pedagogy and technology determine whether a teacher decides to integrate technology in his/her teaching.

Mathematics teaching

Mathematics can be re-organized in schools to provide learners with more conceptual understanding of mathematics (Naidoo, 2012). On one hand, prior 1994, learners were taught very basic concepts preparing them to be unskilled or semi-skilled employees as part of the non-White curriculum. On the other hand, education reformers in South Africa are concerned about South African learners being compared with those of other nations. Majengwa (2010) investigated grade 11 learners‟ understanding of the cosine function with the Geometer‟s Sketchpad. The findings proved the Geometer‟s Sketchpad to be time-saving and helping the learners eradicate misconceptions. The Geometer‟s Sketchpad also helped the learners construct graphs clearly and with ease, to drag and to manipulate the figures. The learners also managed to grasp properties and to understand relationships within the cosine function.

Naidoo and Govender (2014) acknowledged that the use of technology-based tools enriches learning, since visual data promotes and challenges explanation and justification, as opposed to the traditional methods of taking notes, talking and writing on chalkboard. Technology adds more value to the lesson through the use of computer software (Naidoo, 2015). Saha, Ayub & Tarmizi (2010) conducted a quasi-experimental study to examine the effects of using GeoGebra in the learning of Coordinate Geometry among students classified as high visual-spatial ability students (HV) and low visual-spatial ability students (LV). The findings showed that the students‟ performance in learning Coordinate Geometry was enhanced by the use of GeoGebra software. Mathematica is software which has both symbol manipulation capabilities and graphics (Naidoo, 2007). It is usually used for projects in Calculus, though some lecturers prefer the traditional teaching method. Software can be used effectively to address cognitive learning shortcomings by modifying the software such that it caters for average and below average students.

The use of WhatsApp instant messaging is a tool fostered in a social constructivist environment for mathematical learning (Naidoo & Kopung, 2016). The use of technology-based teaching activities saves time during the lecture (Naidoo, 2015). Additionally, using computer programs to construct figures in mathematics prior to the lesson, saves time. Furthermore, the use of technology-based methods helped in making access to abstract concepts and made information more accessible. Govender (2013) used the Geometer‟s Sketchpad Program (GSP) to prove Viviani‟s theorem successfully and with ease.

However, the use of technology-based tools has its challenges. Selvam (2009) stated that technologies are very different in their potential and use. Furthermore, there is very little conviction among policy makers and educationists in most states about the contribution of computers in improving the quality of education. Mathematics teachers ought to be well equipped with the use of technology-based tools so that they are able to explain to learners the integration of technology and Mathematics (Ndlovu, Wessels & De Villiers, 2013).

METHODOLOGY

Four secondary schools which use technology-based tools to teach Euclidean geometry were chosen purposively. The researcher will keep a reflective journal in which to record each and every event regarding data collection. Each of the four Mathematics teachers will be met personally and individually with the aim of acquainting them with her research study topic. In this meeting the researcher will request that each teacher be a participant in her research study. If the teacher agrees to be a participant after the researcher has clarified all ethical issues, the teacher reads and signs the gatekeeper‟s letter for teachers. Thereafter, each of the four Grade 10 Mathematics teachers will complete a questionnaire which will be seeking to elicit which technology-based tools they use in Grade 10 to teach Euclidean Geometry. The learners‟ and parents‟ or legal guardians‟ letters will be given to the teachers. After the learners have agreed to be participants and the parents or legal guardians have agreed that their children be participants in the current study, then data collection process will commence. After a period of three weeks, the researcher will go to the different four schools, to collect the learners‟ and the parents‟ or legal guardians‟ consent letters. The researcher will only start collecting data when it is time to teach Euclidean geometry since the teachers need to adhere to the Annual Teaching Plan. Euclidean Geometry is supposed to be taught from 16 to 29 May in 2018. The researcher therefore needs a time management plan to ensure the observation time(s) do (es) not clash.

Then the researcher will observe any three Grade 10 Euclidean Geometry lessons from each of the four participating secondary schools. Each lesson will be video recorded so as to capture activities which might have been omitted. Then individual face-to-face semi-structured teacher interviews will be conducted. The interviews will be based on the teachers‟ activities whilst they were teaching. Interview responses from each teacher will be audio recorded so as to capture all teachers‟ responses that might have been missed during the interviews. Two focus groups comprising six learners each, will be selected from each school. Each of these two focus groups will be interviewed to triangulate data. Each of the interviews will be audio recorded to capture all the focus groups‟ responses. These responses will be later transcribed and analysed.

Participants and time lines

Four secondary schools in KwaZulu-Natal, South Africa were chosen purposively. The participants will be four Grade 10 Mathematics teachers who use technology-based tools when teaching Grade 10 Euclidean geometry.

From 1 February to 26 March 2018, the researcher met each of the teachers from the four chosen secondary schools with the aim of requesting that they be participants in her research study. Each orientation meeting with the teachers lasted for thirty to forty-five minutes. She will also seek the learners‟, parents‟ or legal guardians‟ permission. The researcher asked the teachers to request the learners on her behalf so as not to disrupt the contact time. The learners‟ and the parents‟ or legal guardians‟ letters were given to the teachers by the researcher to be given to the learners. The researcher will have collected learners‟ and parents‟ or legal guardians‟ gatekeepers‟ letters by 28 April 2018.

By the end of May 2018, all three lessons will have been observed from each participating school. The duration will be fifty minutes or one hour. Teachers‟ individual and learners‟ focus groups interviews should be conducted by 30 June 2018.

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ARE WE WINNING THE BATTLE AGAINST POOR PERFORMANCE IN MATHEMATICS?

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The paper explores and discusses the performance of Grade 7 learners on basic concepts and skills in selected topics on numbers, operations and relationships. 157 learners from 75 schools were assessed. Performance of the learners on some questions of the test was analysed by the learner teachers and the authors of the paper. Typical errors were identified and recommendations for intervention are subsequently offered.

INTRODUCTION AND BACKGROUND TO THE STUDY

The Centre for the Advancement of Science and Mathematics Education (CASME) is a non-profit organisation that was established in 1985. Its vision is "changing the lives of learners through quality and innovative mathematics and science education"; and its mission is "to provide quality teacher professional development opportunities for mathematics and science education in under-resourced and rural schools in South Africa to improve learning outcomes."

CASME thus works with the Department of Education and other stakeholders to support teachers with the teaching and learning of mathematics. We work with teachers across education and training bands. To evaluate the impact of the work that we do, we ensure that learners across the socio-economic spectrum from the schools we support participate in our own Mathematics Olympiads. This initiative enables us to identify gaps in the teaching and learning of selected topics for intervention purposes. The paper seeks to share the identified gaps on a few questions. The paper will also offer recommendations for intervention.

CONCEPTUAL FRAMEWORK

The authors of the paper believe that teaching and learning of mathematics should at all times be driven by, amongst others, conceptual understanding. Van de Walle (2013, p. 1) refers to understanding as "being able to think and act flexibly with a topic or concept". He refers to one hallmark of mathematical understanding as the learner"s "ability to justify why a given mathematical claim or answer is true or why a mathematical rule makes sense". He thus asserts that "understanding must be a primary goal for all of the mathematics" we teach.

Molina (2014, p. 1)) argues that "memorisation of rules and mastery of computations are not the same as true knowledge of mathematical concepts and ideas. The result of these practices is lower achievement in mathematics". He further urges teachers "to shift their instructional focus". He makes a plea that:

> Instead of emphasising misleading educational language, short cuts and memorization of algorithm in math class, we can help students develop a deep understanding of mathematical concepts and identify connections between these concepts (p. 2).

Alfed (n.d., p. 2) describes a person who understands a piece of mathematics as one who can "explain mathematical concepts and facts in terms of simpler concepts and facts". One who can "easily make logical connections between different facts and concepts". A person who recognizes "the connection when you encounter something new (inside or outside of mathematics) that's close to the mathematics" one understands. A person who can "identify the principles in the given piece of mathematics that make everything work".

Finally, Haylock (2008, p. 6) testifies that "all our experience and what we learn from research indicate that learning based on understanding is more enduring, more psychologically satisfying and more useful in practice than learning based mainly on the rehearsal of recipes and routines low in meaningfulness". The above arguments and descriptions of understanding and recommendations informed our study. We strongly believe that learners should be taught mathematics for proficiency; of which conceptual understanding is one of the strands.

LITERATURE REVIEW

While SACMEQ and TIMMS studies do not assess grade 7 learners, their findings have great significance on this study. Both studies maintain that while performance of South African learners is improving, it is still below average. The results of 2014 Annual National Assessment (ANA) are also a witness to this. According to Department of Basic Education (2014, p. 70) report, only 35,4 percent of the Grade 6 learners obtained 50 percent or more in the assessments.

Math Geek Mama (2018, para. 2) identified errors amongst learners which they narrowly categorised into three types, namely, careless errors, computational errors and conceptual errors. Careless errors are those that occur because learners "are not paying attention or they are working too fast" (para. 3). Computational errors are a product of incorrect performance of operations or procedures (para. 5). Conceptual errors are attributed to the misunderstanding of concepts or the use of incorrect logic. Conceptual errors are difficult to identify yet they are the most important "to catch and correct" (para. 7).

Common errors identified by El Paso Community College (n.d. para. 1-4) with the order of operations are:

- Distributing before simplifying inside the parenthesis. For example,

  $3(-4+6\times 2)=12+18\times 6.$
- Incorrectly order of operations within parenthesis. For example,

  $3(-4+6\times 2)=3(2\times 2)$
- Performing multiplication before division. For example,

  $16\div 8\times 2=16\div 16.$
- Performing addition before subtraction. For example,

  $12-5+4=12-9.$

With regards to the exponents, the most common mistake is multiplication of the base by the exponent. For example, $3^2=3\times 2$. Incorrect application of exponential rules is also rife. For example, $\sqrt{64+36}=\sqrt{64}+\sqrt{36}$ and $\sqrt{9^{16}}=3^4$. Unfortunately, this paper is unable to exhaust errors identified by the research studies.

DATA COLLECTION

A sample of 157 Grade 7 learners from 78 schools from schools in Southern and Central KZN schools was used for the study. Each school was given a selection test. They had to use it to prepare and select learners for the second phase of our Olympiads. Each school had to identify (at most) the top 2 learners to participate.

While the assessment covered all content areas, this paper glimpses into the performance in the multiple choice questions on numbers, operations and relationships only. The test items used in the tool were taken and adapted from previous Olympiads papers; TIMMS and SACMEQ tools; and a variety of worksheets that are freely available on the web.

Delimitations

While the item analysis of the learner performance was done with the teachers, learners were not interviewed to validate the tool used to collect data. The authors of the paper and the teachers could not come to a consensus for selection of some distractors as they also could not establish rationale for them.

DATA ANALYSIS AND FINDINGS

Performance in 1.1.1

The question was:

Which of the numbers below is the correct answer for $10 \times 2 + (6 - 4) \div 2$?

A. 21 B. 20 C. 12 D. 11

The correct option is A. The graph below shows that about 66 percent of the learners chose option D?

$n = 157$

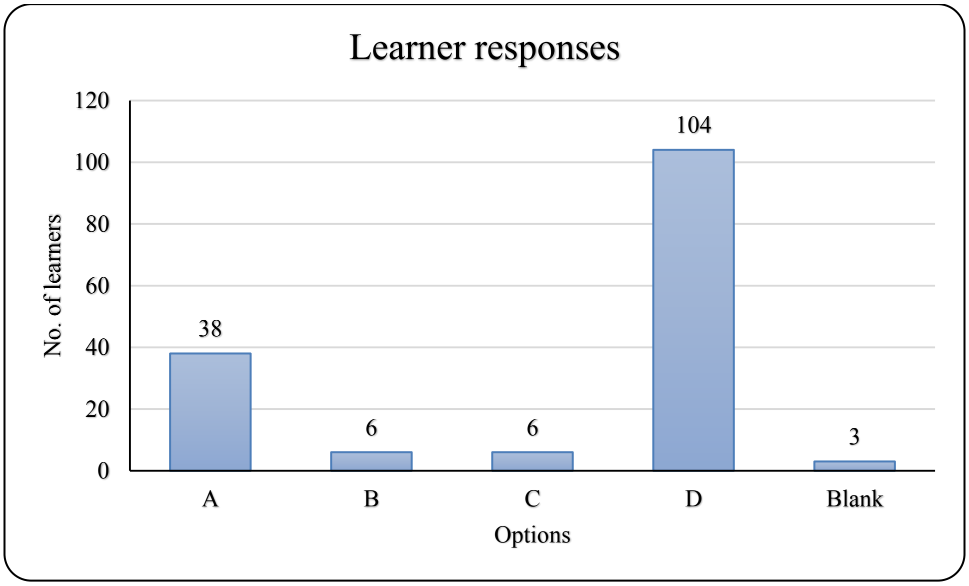


Figure A

The question assesses if learners know the conventional order of operations. The overwhelming choice of option D shows that learners still experience a challenge with the order of operations. The choice of the option indicates that learners worked from the left to the right. That is, they multiplied 10 by 2 to get 20, added 6 to get 26, subtracted 4 from 26 to get 22, and divided 22 by 2 to get 11, which is option D.

Performance in 1.1.2

The question was:

Which of the numbers below is the **Lowest Common Multiple** of 12 and 18?

A.3 B. 6 C. 18 D. 36

The correct option is D. While the majority of learners chose the correct option, a significant proportion of learners (about 35 percent) chose option A.

$n = 157$

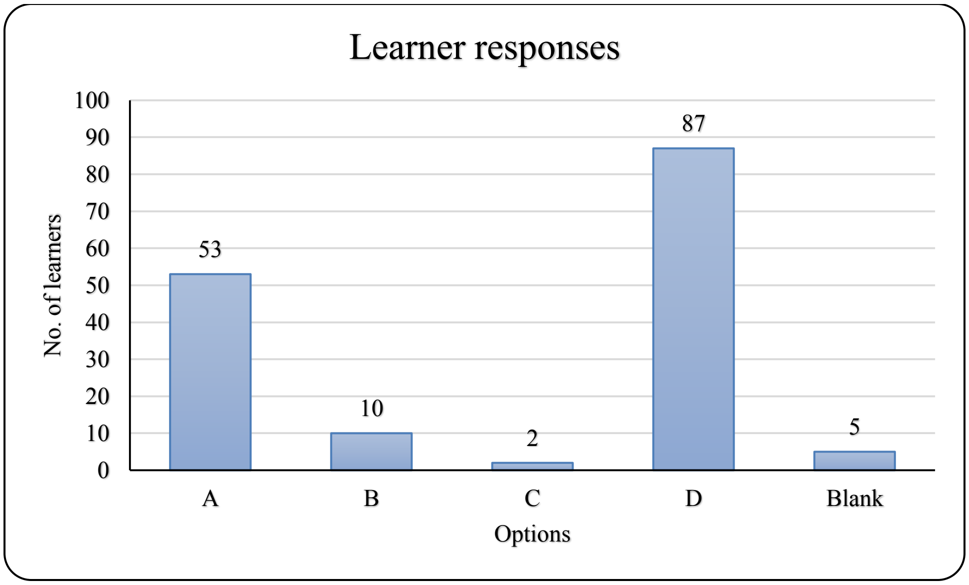


Figure B

Probably, learners still confuse multiples and factors. The selection of the option could probably mean that learners regarded 3 as the lowest common factor of 12 and 18.

Performance in 1.1.3

The question was:

> Last year there were 1 172 students at Ahoy Primary School. This year there are 20 percent more students than last year. Approximately how many students are at Ahoy Primary School this year?
>
> A. 1 200 B. 1 400 C. 1 600 D. 1 800

The correct option is B. Performance in this question was not bad, but the number of learners that chose A or D is worrying (see Table 1 below).

$n = 157$

Option	A	B	C	D	Blank
No. of learners	32	66	18	23	18

Table A

Learners who chose A may have given 1 172 rounded off to the nearest 100. Also of concern is the number of learners who did not respond to the question (more than 10 percent).

Performance in 1.1.4

The question was:

Which of the numbers below is equivalent to 4^3?

A. 7 B. 12 C. 64 D.81

The correct option is C. Performance in the question was relatively good (refer to Table 2 below).

$n = 157$

Option	A	B	C	D	Blank
No. of learners	4	24	123	1	5

Table B

While about 78 percent of the learners were able to choose the correct answer, it is worrying that 24 learners chose option B as this indicates that they multiplied the base by the exponent.

Performance in 1.1.5

The question was:

Each figure represents a fraction.

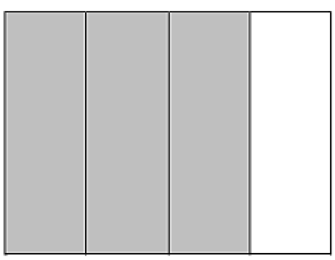


Figure 1

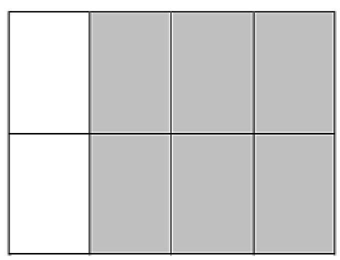


Figure 2

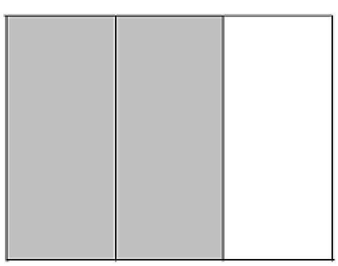


Figure 3

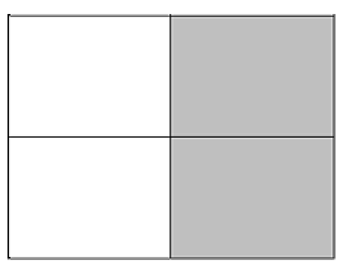


Figure 4

Which two figures represent the same fraction?

A. 1 and 2 B. 1 and 4 C. 2 and 3 D. 3 and 4

The correct option is A, which the majority of learners (more than 70 percent) chose.

$n = 157$

Option	A	B	C	D	Blank
No. of learners	112	17	11	15	2

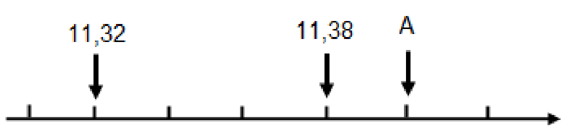
Table C

This is probably an indication that learners have no problem with the equivalent fractions.

Performance in 1.1.6

The question was:

Which number is equal to A in the number line below?



A. 11,39 B. 11,4 C. 12 D. 12,32

The correct option is B. The majority of learners chose the correct option.

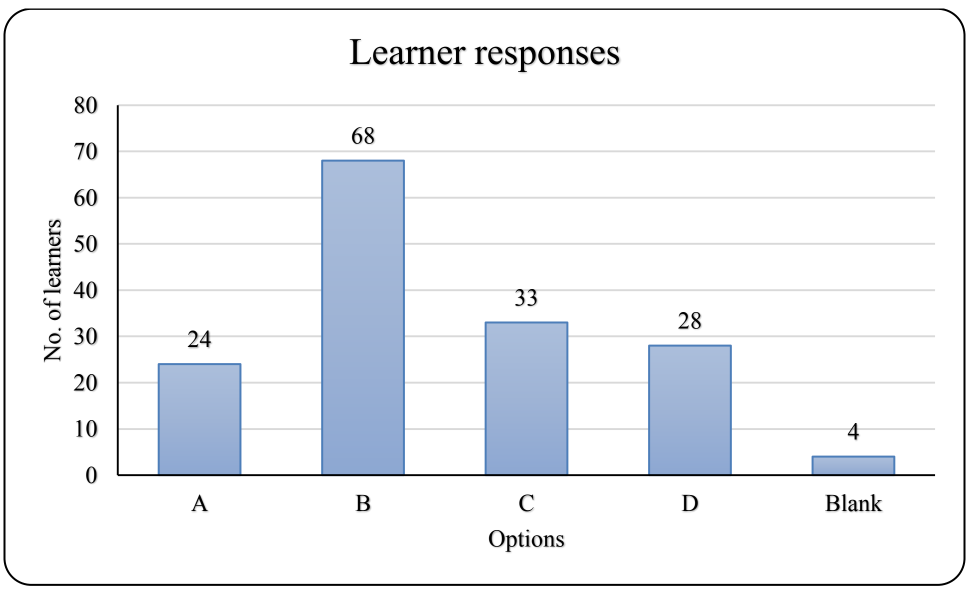


Figure C

However, more than 50 percent of the learners (75 learners) chose options A, C or D. This is an indicator that something needs to be done regarding ordering and comparing of decimals.

Performance in 1.1.7

The question was:

> A stack of 200 identical sheets of paper is 2,5 cm thick. What is the thickness of one sheet of paper?
>
> A. 0,008 cm B. 0,0125 cm C. 0,05 cm D. 0,08 cm

The correct option is B, but the majority of learners chose option C.

$n = 157$

Option	A	B	C	D	Blank
No. of learners	25	35	48	30	19

Table D

The distrators in this question could not help us identify the problem. We could only speculate from the spread of data that learners were guessing, which is one problem with multiple choice questions. Also striking here is the number learners who did not chhose any of the options (More that 10 percent).

Learner performance in 1.1.8

The question was:

> A fifth is tenth of a number. What is the number?
>
> A. 2 B. 1 C. $\frac{1}{5}$ D. $\frac{1}{10}$

Correct option is A, but the popular option was C.

$n = 157$

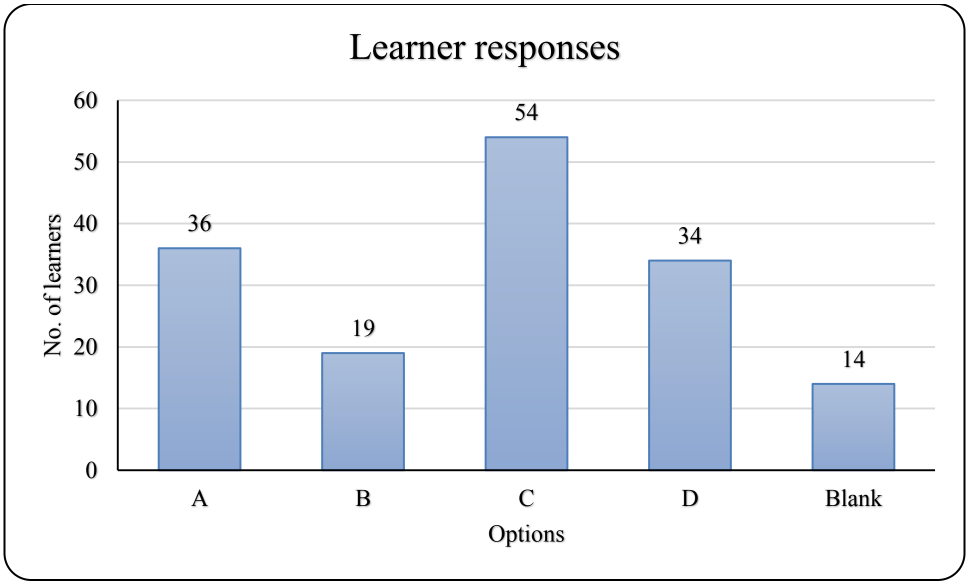


Figure D

This could be attributed to the fact that learners do not understand word problems. They just saw the word “fifth” in the question and went for option C. The same reason could be attributed to the choice of option D.

Performance in 1.1.9

The question was:

A class has 28 students. The ratio of girls to boys is 3 : 4.
How many girls are in the class?

A. 21 B. 16 C. 15 D. 12

Correct option is D and most learners got this one.

$n = 157$

Option	A	B	C	D	Blank
No. of learners	25	23	9	90	10

Table E

However, the choice of options A and B was somehow a concern as learners multiplied by 3/4 and 4/3 respectively.

Performance in 1.1 10

The question was:

Asanda received R1 200 in cash for mowing lawns. She put 20% into a bank account, spent ¼ of the amount and kept the rest in cash. How much did she keep in cash?

A. R540 B. R660 C. R675 D. R720

The correct option is B yet the majority of learners chose option D.

$n = 157$

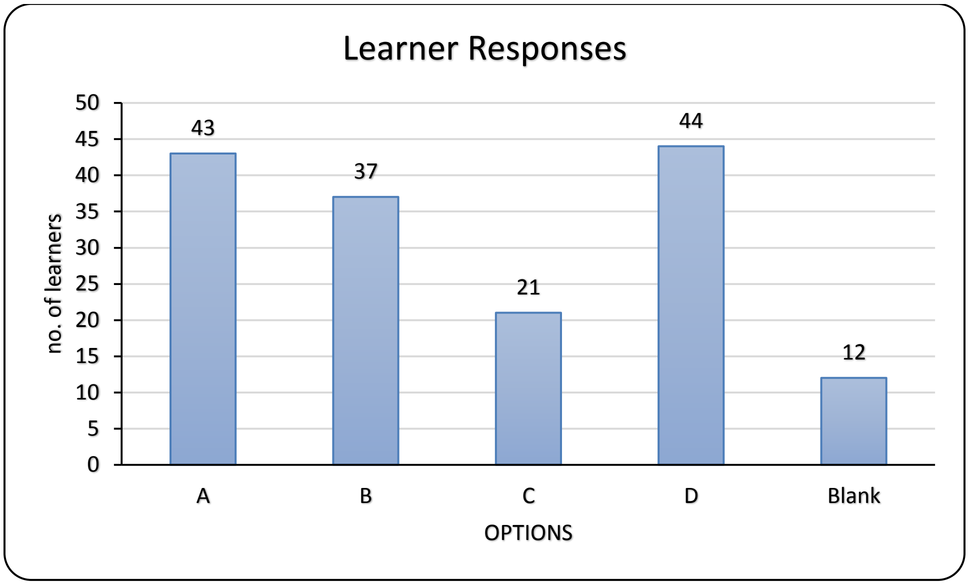


Figure E

The choice of option D could be attributed to misinterpretation of the question. Learners calculated 20% of R1 200 and got R240. They subtracted R240 from R1 200 and then calculated 1/4 of the remaining amount.

RECOMMENDATIONS AND CONCLUSION

Math Geek Mama (2018, para. 1) points out that "making mistakes in math is a good thing and can help kids learn and understand more deeply". Lannin, Barker and Townsend (2007, p. 44) describe errors from a quotation by Salvador Dali "as opportunities for deepening one‟s understanding and as important components of learning process". They (Lannin et al., 2007) then argue that "the view of errors as a vehicle for learning, rather than an activity to eradicate, continues to gain momentum in mathematics education". We thus also wish to recommend use of identified errors to teach for understanding.

Learners should be taught the order of operations in context. They should be taught such that they can differentiate between factors and multiples. For the latter, CPALMS (2017, p. 3) recommends asking questions that elicit thinking. For example, "what does it mean to be a multiple of a number? Can you give me a multiple of 10? What does it mean to be a factor of a number? Can you give me a factor of 10? How does a factor differ from a multiple?" Learners should be taught mathematical notations as part of mathematical language. As Molina (2014) correctly suggests, mathematics instruction should shift its focus from rote learning of mathematics to teaching for conceptual understanding.

While the item analysis above indicates gaps in teaching and learning, one also could confidently claim that strides have been made towards winning the battle against poor performance. While the participants showed problems with the order of operation, factors and multiples, as well as word problems; they did very well in fractions, ratio and exponents. As mathematics teachers, we thus have to remain optimistic about the future of learner performance in mathematics.

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TEACHING AND LEARNING IN MULTILINGUAL MATHEMATICS CLASSROOMS THROUGH THE USE OF AN INSTRUCTIONAL MATHEMATICS APPLICATION PROGRAMME

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Mathematics classrooms are often characterized by various teaching aids including, more recently, handheld devices that are often loaded with Mathematics Application (App) so as to provide assistance in enhancing learners' Mathematical understanding. However, the very same Mathematics App that is meant to aid the learner into more understanding can be a hindrance if the Language of Learning and Teaching (LoLT) is not carefully considered in the App design stage. This study aims at identifying what language intricacies might exist that currently could be overlooked by focusing on one Mathematics Application called onebillion©.

INTRODUCTION

Studies have shown that early mathematics learning and reading skills are a great predictor of later achievement in the learner‟s academic life (Duncan et.al, 2007). The Curriculum and Assessment Statement (CAPS) (2011) policy document in South Africa emphasises that "Foundation Phase Mathematics forges the link between the child‟s pre-school life and life outside school on the one hand, and the abstract Mathematics of the later grades on the other hand. In the early grades children should be exposed to mathematical experiences that give them many opportunities "to do, talk and record" their mathematical thinking". From these views, it can be argued that both views imply that if the learner has a solid understanding of mathematics in the early school years, the chances are that this would positively influence later performance in mathematics. This implies that the inverse is also possible. One of the key ingredients to successful mathematics learning is communication within the mathematics classroom. Teachers, the textbooks they use, and the mathematical programmes they employ in the course of teaching, all need to communicate mathematical ideas to learners and learners need to understand and communicate back their understanding. There are many ways that this communication can take place, however one integral part which is essential for successful communication, is language. The connection between mathematics and language cannot be ignored as mathematics is taught in and through language, and especially so in the context of South Africa where multilingualism is the order of the day (Barwell, 2009; Boulet, 2007; Pimm, 1981)

Barwell (2009) argues that the learner‟s proficiency, or lack thereof, in the language of learning and teaching (LoLT) plays a major role in their mathematics performance, compared to their peers who are monolingual and as such socially and academically speak and operate in the same language. This emphasises the role that language plays in the everyday teaching and learning of Mathematics. Apart from the multilingual contexts that characterise the everyday teaching and learning, the current mathematics classrooms is also characterised by the use of various teaching aids to help learners understand certain mathematical concepts and develop a deeper conceptual understanding (Jenkins, 1957). At times, technological Applications are used in the classroom for the same aim, to reinforce mathematical concepts and aid understanding (Ferrini-Mundy & Breaux, 2008)

For the purpose of this research, I intend to investigate how the use of a Mathematics App called onebillion© enables the teaching and learning of mathematics in classroom situations. One way in which this has been achieved is by the home language provision that is offered by the App in order to enhance mathematics access to the learners in a certain social context who are not English first language speakers. Roblyer and Doering (2013) identify the six principles that ought to be present in every school mathematics program. These are as prescribed by the National Council of teachers of Mathematics (NCTM) namely, equity, Curriculum, teaching, learning, assessment and Technology. Focusing on the last item which is technology, Roblyer and Doering (2013) argue that technology can enable the teacher to move towards a more learner centered approach and thus allow for context to be embraced in the mathematics classroom. Furthermore, they argue that technology can enable the learner to see mathematics in a less abstract way and rather see and experience mathematics in a more concrete manner and this is most applicable in the elementary school level. Technology affords what they term virtual manipulatives which can be manipulated as need be. Sharples et al. (2016) in their study of Mobile learning argue that computers and mobile phones (including tablets and iPads) are tools that assist the learners in the process of acquiring knowledge.

South Africa is one context which has historically been affected by the plague of Apartheid, with effects that are still being felt by the country up to this day (Phakeng & Essien, 2016). In recognition of the necessity to ensure equal access to education for all, the language in education policy (1996) has made provision for foundation phase learners to receive instruction in their mother tongue supporting conceptual growth as well as ensuring that there is a continuity between the leaners home language and the language of learning and teaching (LoLT). The South African School Act under Language in Education Policy allows for a learner to choose the language of teaching when applying for schools, keeping in mind that there are a number of other factors that need be taken into account in this regards. However, what this means is that the learner is allowed to learn in his/her mother tongue throughout primary and high school years (Schools: Law and Governance, 2014). Despite this allowance there is a prominent trend in which the first three years (Grade 1 – 3) learners learn and are taught in their home/first language, however in Grade 4 the LoLT changes into English (Manyike, 2013).

In this study, the mathematics App that I will base my study upon is one which has been developed by onebillion©. This App focuses on core mathematics concepts for the first 4 years of schooling (Grade R to 3 in App store and Google play in a variety of languages (onebillion©, 2018)). The App is currently being piloted in a Province in South Africa and being used by Grade 1 learners. The App was originally developed in English language and has now been translated into African languages (isiZulu in the case of the current study).

Translating the mathematics register from one language to another (English to isiZulu in the case of this current study) is not a straightforward enterprise especially when one language (English) has a long tradition of being used in mathematics (and so has well developed mathematics register) while the other language (isiZulu) is still at a developing stage in terms of the mathematics register. The extent to which language issues can be found in Mathematics Applications that have been translated from a developed language to a developing language has not been an explicit focus in research. This study sets out to investigate this phenomenon. To achieve this, I will focus on the onebillion© Mathematics App that is offered in IsiZulu centering on the Mathematical language/register used in the App. This study is therefore informed by the research questions below:

1. How does the IsiZulu mathematics language in the onebillion© App compare to the language found in the Curriculum used by the teachers and the learners?
2. What is the teachers‟ perception of the IsiZulu as it is used by the App?
3. What has the App enabled the teachers and learners to understand better?
4. What language issues are imbedded in the use of the onebillion© App?

Together these questions will help me map the mathematics App to the Curriculum and highlight the teachers‟ view of the language as used in the App which in turn could have implications on the mathematical understanding of the learner.

SIGNIFICANCE OF THE STUDY

The contribution this research will make is that, whilst mathematical content is often critically scrutinised before a roll out of an intervention, it is important that we do not overlook the role which language can play in ensuring that the learning that is offered to the learner is not diluted or impeded simply by overlooking the importance of language when an App is being designed.

THEORETICAL FRAMEWORK

The theoretical framework that will be used to inform this study will be that of Engelström (1999) Activity Theory. This theory has its etymology in Vygotsky‟s (1978) work on mediation, which spoke to how leaning was mediated by cultural signs and tools. Engelström‟s (1999) Activity theory adds unto the mediation theory of Vygotsky, the work of Leontev on action and activity highlighting division of labour, and constructs a system that takes into account the object, subject, mediating artefacts, rules, community and division of labour as concepts that all play part in creating, shaping and enabling the learning space and as such supporting a learner in attaining new knowledge.

In the context of this study, the subject is the learner, who exists and is influenced by the community (the larger community, the school and the mathematics classroom) s/he is situated in and the rules (how thigs are done in the classroom) that governs this community in this case the mathematics classroom, in which there is division of labour highlighting the roles which the teacher and the learners are expected to perform, to ensure that the learners‟ enhances his/her knowledge and skills (object), resulting in change of the learners‟ knowledge thus the desired outcome (changed object). Furthermore, the mediating artefacts in this study are represent the language and technology that is present in this context which, when well-constructed, can enhance the teaching and learning of mathematics. Thus this theory allows for me to zoom into the language and technology (mediating artefacts), whilst taking into account other concepts that are at play in forming the complex teaching and learning environment.

METHODOLOGY

Yin (2016) argues that Qualitative research affords the researcher the ability to conduct an in-depth study of the research participants in their everyday context. This study aims to explore individual‟s rich descriptions of their own experience in relation to the Mathematics App onebillion©. In this instance the object of study is thus the onebillion© App and as such the Case Study research approach will be adopted for this research as it will allow for in depth description and understanding of the language nuances associated with the App.

In accessing the rich data and taking into account the complex contexts that the participants are situated in, semi-structured interviews and classroom observations will be the preferred data collection method (Yin, 2016). Two teachers who assist the Grade 1 learners with the App will be interviewed in this study as well as 7 learners who will be randomly selected will be interviewed also. Thus the teachers and the learners will be my sample.

In terms of observation 2, classrooms will be observed in the following ways. Firstly, I will observe the classroom language usage between learners and teachers in the traditional classroom setting, and secondly I shall observe the same classes in the mobile classroom where the learners engage with the mathematics App. In addition to the interviews and observations to be conducted, I will be looking at the isiZulu Language that has been utilised in the App with a focus on the Mathematical Register. At the time of the presentation, I would have collected data and started with data analysis. It is therefore my hope that I will be able to present preliminary findings from my study.

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DEVELOPING MATHEMATICAL REASONING IN THE THRESHOLD OF LEARNERS' ERRORS AND MISCONCEPTIONS

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This paper is written in a constructivist framework presuming that learners use their prior knowledge to receive and give meaning to the new mathematical knowledge they learn. In the study, grade 9 learners' errors and misconceptions on the topic exponents are discussed. Some errors shown include cancellation errors as well as confusion with the laws of exponents. The empirical data is used as a basis to develop mathematical reasoning in the learners.

BACKGROUND TO THE STUDY

This article discusses the errors and misconceptions unveiled in grade 9 learners‟ work on the topic of exponents. It discusses how mathematical reasoning can be developed through shown learner errors and misconceptions. Errors are wrong answers due to planning, they are systematic in that they are applied regularly in the same circumstances‟ (Olivier, 1989, p 3). To Olivier, "errors are symptoms of the underlying conceptual structures that are the cause of errors". The above suggests that that errors and misconceptions are related to each other but different. The underlying conceptual framework that causes errors is called a misconception (Nesher, 1987). Furthermore, Nesher argues that misconceptions lead to a cluster of errors, which are not sporadic. However, we need to establish the cause of misconceptions which give rise to errors. Nesher argues the expertise of learners is to make errors which must always be welcome in the classroom. They show the incompleteness of learners‟ knowledge and are often milestones of learning. Indeed, any detection of errors, raises doubts in our beliefs and leads to growth of valid knowledge. Indeed, when we realize that our concepts are in disharmony with our environment, we are in a state of tension and dissonance - a state of cognitive conflict and we are bound to act, we learn in order to regain psychological equilibrium.

In constructivism, knowledge does not simply arise from experience. Rather, it arises from the interaction between experience and our current knowledge structures(Olivier,1989). The above imply that learners are not seen as passive recipients of knowledge from the environment, rather they will be constructing their own knowledge through assimilation and accommodation (Hatano ,1996, p.202). When a new idea can be incorporated directly into an existing schema we call that assimilation. The new idea is interpreted or re-cognised in terms of an existing schema. Accommodation is when idea may be quite different from the learners" existing knowledge. In such a case it is necessary to re-construct and re-organise the schema. In trying to receive knew knowledge; learners usually end up overgeneralizing the previous knowledge to the new concept which is quite different. It is this overgeneralisation which may lead to misconceptions. The notion of misconception denotes a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic, unconnected and non-systematic errors (Nesher, 1987, p.35). Nesher further argues that misconceptions have their roots in earlier learned meaning systems, they could be derived from previous instruction. On the same note (Luneta and Makonye,2010) argue that "errors and misconceptions are related but they are different. Accordingly, they say an error is "a mistake, a slip, blunder or inaccuracy and a deviation from accuracy" (ibid, p.35).

Luneta and Makonye further argue that errors are visible in learners" artifacts such as written text and conversations whereas misconceptions can be hidden in correct answers (Smith, DiSessa& Roschelle1993) when correct answers are accidental.

METHODOLOGY AND DATA ANALYSIS

The following questions are the few questions selected for error and misconception analysis from ten questions given to grade 9 learners at Secondary School in South Johannesburg.

Question 1: Simplify (i) 2^2 (ii) 3^2

All learners got the correct answering part (i) $2^2 = 2 \times 2 = 4$. Quite a number in one of the classes got part (ii) $3^2 = 3 \times 2 = 6$, wrong which clearly indicates that a wrong concept was applied to part (i) yielding a "correct" answer. If part (ii) had not been given the misconception would have remained hidden in the correct answer to part (i). When some learners were asked to explain their answer they said they multiply the base and the power.

Question 4: Simplify $\frac{x^{10}}{x^{14}}$

Two learners gave the answer as $\frac{x^{10}}{x^{14}} = \frac{10}{14} = \frac{5}{7}$

The above response clearly shows a faulty understanding of the laws of exponents. The learner did not understand the question. Learner confused exponents with algebraic fractions where one can simplify by identifying the HCF of both numerator and denominator and reduce to simplest form. To the learner x is the HCF yielding $\frac{10}{14} = \frac{5}{7}$ when simplified. This is clearly a misconception.

Question 5: Simplify the following: $(x^2)^3$.

The following answer was observed: $(x^2)^3 = x^{2+3} = x^6$

The learner in the answer above clearly showed an error in writing x^{2+3} instead of $x^{2\times3}$. However, the learner in due process wrote x^6, the correct answer, suggesting that the error was a writing one while the application of the rules of exponents were applied correctly.

Question 6: Evaluate $(2x^2)^3$.

A number of learners wrote $2x^{2\times3}$ yielding $2x^{2\times3} = 2x^6$.

The answer shows a wrong application of the rules of exponents. The learners concentrated on the variable and forgot to distribute the index to both exponents in the bracket. This is a misconception. Learners should have written $(2x^2)^3 = 8x^6$.

DISCUSSIONS AND CONCLUSIONS

Learners‟ errors and misconceptions play an important role in developing mathematical reasoning in exponents. Mathematical reasoning is the critical skill that enables a learner to make use of all other mathematical skills and make connections between concepts, procedures in a problem solving milieu. Learners errors‟ and misconceptions play a dual function, first for the learner in that the learner will realise where they went wrong and for the teacher to detect what the learner lacks in terms of knowledge. This is supported by Nesher (1987, p.35) who argues that committing an error reveals the incompleteness of learners‟ knowledge and enables the teacher to contribute additional knowledge, or lead them to realise for themselves where they went wrong. When learners make errors and misconceptions the discerning teacher can look for the underlying principles leading to the same and with appropriate intervention can bridge the gap between the new knowledge and the learner‟s current schema. When an erroneous principle is detected at this deeper level it can explain not only a single, but a whole cluster of errors.

However, misconceptions are hard to detect because in some occasions the mistaken rule is disguised by a correct answer, that is a student may get the "right "answer for the wrong reasons. Learners errors and misconceptions can help teacher to design learning instruction that uses learners‟ prior knowledge as the spring board from which new knowledge can be acquired. We must know how this new knowledge is embedded in a larger meaning system that the child already holds and from which he derives his guiding principles (Nesher, 1987, p.36).

Also, it is of great importance that the teacher handles learners‟ errors and misconceptions carefully because through the same the learner can be encouraged to learn new knowledge or be discouraged to learn. It is crucial to know specifically how the already-known procedure may interfere with material now being learned (Nesher, 1987 ibid).

When designing instructions, the teacher should mark clearly similarities and differences between the new elements to be learned and the old knowledge that the learner holds. All the new elements, which resemble but differ from the old ones, should be clearly discriminated in the process of instruction and the teacher should expect to find errors on these elements (Nesher, 1987). The teacher should allow learners to make mistakes as it is from the same errors that learners acquire knowledge.

Errors and misconceptions are seen as the natural result of children"s efforts to construct their own knowledge and these misconceptions are intelligent constructions based on correct or incomplete (but not wrong) previous knowledge. Errors and misconceptions should not be treated as terrible things to be uprooted, since this may confuse the child and shake his confidence in his previous knowledge (Nesher, 1987). Teachers should create chances that would enhance learning. Teaching methods that allow learners" active participation in the learning process must be effectively employed. Nesher (1987) and Olivier (1988) argue that discussion, communication, reflection and negotiation of meaning are essential features of a successful approach to resolve pupils" misconceptions.

From the above discussion it is evident that different teaching approaches should be adopted so as to minimize learners" errors and misconceptions. Also it has been shown that learners" errors and misconceptions should be treated with great care since it is from learners" errors and misconceptions that teachers get to know how learners are trying to construct their own knowledge.

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ADDITIVE RELATION WORD PROBLEMS IN THE DBE WORKBOOKS

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Workbooks were introduced by South African Department of Basic Education (DBE) in 2011. Although the workbooks were designed as supplementary materials, in some schools they are used as the sole teaching text. Because of this, and because they represent an exemplification of the curriculum by the DBE, an analysis of the content coverage of the workbooks is warranted.

This paper provides such an analysis in terms of additive relation word problems. This was done using a comprehensive typology of additive relation word problem types based on typologies used in previous studies. All the additive relation word problems in the 2017 Grade 1 to 3 foundation phase mathematics workbooks were categorized according to this comprehensive typology.

In total there were 61 single step additive relation word problems with numerical answers across the three grades. This is a small number in comparison to textbooks used in other countries. There is an uneven distribution of problem types with 40 change type problems, no equalize problems, 18 collection problems and only three compare type problems. There are also more problems in the easier subcategories and few or no problems in the more difficult subcategories.

This paper provides evidence for the need to revise the word problems in the DBE workbooks - both in terms of the number of problems and the distribution of the problem types. It also provides a theoretical framework to use in the revision of the workbooks and in any supplementary teaching material developed for teachers, and makes specific recommendations in terms of a more preferable distribution of word problems.

INTRODUCTION

Since 2011 the South African Department of Basic Education (DBE) has provided all Grade 1-6 learners in public schools with language, mathematics and life skills workbooks. Although the workbooks were designed as supplementary materials, in some schools they are used as the sole teaching text. Because of this, and because they represent an exemplification of the curriculum by the DBE, an analysis of the content coverage of the workbooks is warranted.

This study provides such an analysis in relation to one aspect of foundation phase mathematics, namely word problems. Word problems are a key aspect of early grade mathematics curriculum, internationally and in South Africa (Department of Basic Education, 2011). Three diagnostic reports on the South African Annual National Assessments (ANA) have identified word problems as a recurring weakness (Department of Basic Education, 2012, 2014, 2015) in the foundation phase, thus warranting further research.

In Grades 1 to 3 word problems focus on additive relations and multiplicative reasoning. Various mathematics education researchers have invested time in classifying additive relations word problems and developing typologies against which a curriculum or text could be evaluated. In this study a comprehensive typology, combining earlier typologies, is used to analyse the prevalence or absence of particular word problem types in the DBE foundation phase mathematics workbooks.

THEORETICAL FRAMEWORK

Since the late 1970s, researchers have developed classifications of additive relation word problems. For this study, two of the original typologies (Carpenter, Hiebert, & Moser, 1981; Riley, Greeno, & Heller, 1984) were combined to develop a comprehensive typology. The comprehensive typology consists of four main categories, including the equalise category which was present in early typologies but has been excluded from later ones.

Table 1: Exemplification of comprehensive typology

	Change			
	3 + 5 = 8	Increase	8 – 5 = 3	Decrease
Result unknown	3 + 5 = [ ]	Nosisi had 3 marbles. Then she got 5 more marbles. How many marbles does she have now?	8 – 5 = [ ]	Nosis had 8 marbles. Then she lost 5 marbles. How many marbles does she have now?
Change unknown	3 + [ ] = 8	Nosisi had 3 marbles. Then she got **some** more marbles. Now Nosisi has 8 marbles. How many marbles did she get?	8 – [ ] = 3	Nosisi had 8 marbles. Then she lost **some** marbles. Now Nosisi has 3 marbles. How many marbles did she loose?
Start unknown	[ ] + 5 = 8	Nosisi had **some** marbles. Then she got 5 more marbles. Now she has 8 marbles. How many marbles did she have at first?	[ ] – 5 = 3	Nosisi had **some** marbles. Then she lost 5 marbles. Now she has 3 marbles. How many marbles did she have at first?

	Equalise			
	3 + 5 = 8	Increase	8 – 5 = 3	Decrease
Target unknown	3 + 5 = [ ]	Nosisi has 3 marbles. If she wins 5 marbles she will have the same number of marbles as Silo. How many marbles does Silo have?	8 – 5 = [ ]	Nosisi has 8 marbles. If she loses 5 marbles she will have the same number of marbles as Silo. How many marbles does Silo have?
Change unknown	3 + [ ] = 8	Nosisi has 3 marbles. Silo has 8 marbles. How many marbles does Nosisi have to win to have the same number of marbles as Silo?	8 – [ ] = 3	Nosisi has 8 marbles. Silo has 3 marbles. How many marbles does Nosisi have to lose to have the same number of marbles as Silo?
Start unknown	[ ] + 5 = 8	Silo has 8 marbles. If Nosisi wins 5 marbles she will have the same number of marbles as Silo. How many marbles does Nosisi have?	[ ] – 5 = 3	Silo has 3 marbles. If Nosisi loses 5 marbles she will have the same number of marbles as Silo. How many marbles does Nosisi have?

	Collection			
	3 + 5 = 8	Different attributes	3 + 5 = 8	Different ownership
Collection unknown	3 + 5 = [ ]	Nosisi has 3 red marbles and 5 blue marbles. How many marbles does he have altogether?	3 + 5 = [ ]	Nosisi has 3 marbles. Silo has 5 marbles. How many marbles doe they have altogether?
Subset unknown	3 + [ ] = 8	Nosisi has 8 marbles. 3 are red and the rest are blue. How many blue marbles does he have?	3 + [ ] = 8	Nosisi and Silo have 8 marbles altogether. Nosisi has 3 marbles. How many marbles does Silo have?

	Compare			
	3 + 5 = 8	More than	8 – 5 = 3	Fewer than
Compared quantity unknown	3 + 5 = [ ]	Silo has 3 marbles. Nosisi has 5 **more** marbles **than Silo**. How many marbles does Nosisi have?	8 – 5 = [ ]	Silo has 8 marbles. Nosisi has 5 **fewer** marbles **than Silo**. How many marbles does Nosisi have?
Difference unknown	3 + [ ] = 8	Nosisi has 8 marbles. Silo has 3 marbles. How many marbles does Nosisi have **more than Silo**?	8 – [ ] = 3	Nosisi has 3 marbles. Silo has 8 marbles. How many marbles does Nosisi have **fewer than Silo**?
Referent	[ ] + 5 = 8	Nosisi has 8 marbles. She has	[ ] – 5 = 3	Nosisi has 3 marbles. She has 5

unknown	5 **more** marbles **than Silo.** How many marbles does Silo have?	**fewer** marbles **than Silo**. How many marbles does Silo have?

METHODOLOGY

The comprehensive typology was used to do an analysis of the prevalence and distribution of additive relation word problems in the Grade 1 to 3 DBE mathematics workbooks, against a specific typology of word problems. The 2017 versions of the DBE foundation phase mathematics workbooks were selected. The workbooks were selected because in the two schools forming part of the bigger research project, the workbooks were the primary (and in some cases the sole) teaching text used by the foundation phase teachers in 2017.

Electronic version of the 12 (4 terms per grade across 3 grades) DBE workbooks were used to identify all additive relation word problems. First each page examined for word problems and a copy of the relevant pages was extracted from the electronic document. As in Stigler et al.'s study (1986, pg. 160), a problem had to present two premises and a question in order to be included. Word problems that required two steps were excluded (e.g. I bought 15 sweets. I ate 2. I gave my friend 4. How many sweets do I have left?). Almost all of the two-step problems had money (4 out of 8) or weight as a context (3 out of 8). Only one had sweets as a context. Word problems that did not require a numerical answer were also excluded (e.g. I have a R5 and a R1 coin. My friend has three R5 coins. Who has the most money?).

Each word problem was captured on an excel sheet and classified according to the three levels of the comprehensive typology. The initial classification was done by the author. The list of word problems together with the typology was then sent to a colleague who independently classified the 61 problems. Any cases where there were differences were discussed until an agreement was reached.

Because this was a text-based study, no ethical approval was necessary.

RESULTS

The frequency of each of the 22 problem types in the comprehensive typology, by grade, is shown in Table 2. The first interesting thing to note from the table is the very small total number of word problems (61) that appear in the workbooks across the three grades. This small number is not problematic if the workbooks are being used as they were designed to be used – namely as supplementary resource for learners to practice what they have been taught in class. However, when they are being used as the sole teaching resource and learners not being exposed to any other word problems, the very small number of problems is a matter for concern.

Table 2: Frequency of different types of word problems, by grade

	Gr 1	Gr 2	Gr 3	Gr 1	Gr 2	Gr 3	Total
	Change						
	Increase			**Decrease**			**Total**
Result unknown	7	4	4	5	7	0	27
Change unknown	0	0	8	2	0	3	13
Start unknown	0	0	0	0	0	0	0
	Equalise						
	Increase			**Decrease**			
Target unknown	0	0	0	0	0	0	0
Change unknown	0	0	0	0	0	0	0
Referent unknown	0	0	0	0	0	0	0
	Collection						
	Attributes			**Ownership**			
Collection unknown	1	0	2	6	3	1	13
Subset unknown	1	4	0	0	0	0	5
	Compare						
	More than			**Fewer than**			
Compared quantity unknown	0	0	0	0	0	1	1
Difference unknown	1	0	1	0	0	0	2
Referent unknown	0	0	0	0	0	0	0

Looking at the distribution of word problems across the four main problem types reveals a greater focus on change type problems, no equalise type problems and very few compare type problems. The under representation of collection and compare type problems (18 and 3 respectively) is a reason for concern as these two-word problem types provide learners with an opportunity to engage with useful mathematical concepts.

In terms of the distribution across the grades (see Figure 1), the total number of word problems varies across the grades from 23 in Grade 1, to 18 in Grade 2 to 20 in Grade 3. The number of word problems in each category also varies across the grades with Grade 1 and 3 having more change and compare type problems than

Grade 2 and the number of collection type problems decreasing with each grade. A more preferable distribution would include more problems in each grade, some equalise problems (particularly in Grade 1 and 2) and a decrease in change type problems mirrored by an increase in compare type problems (which are conceptually more challenging for learners) across the three grades.

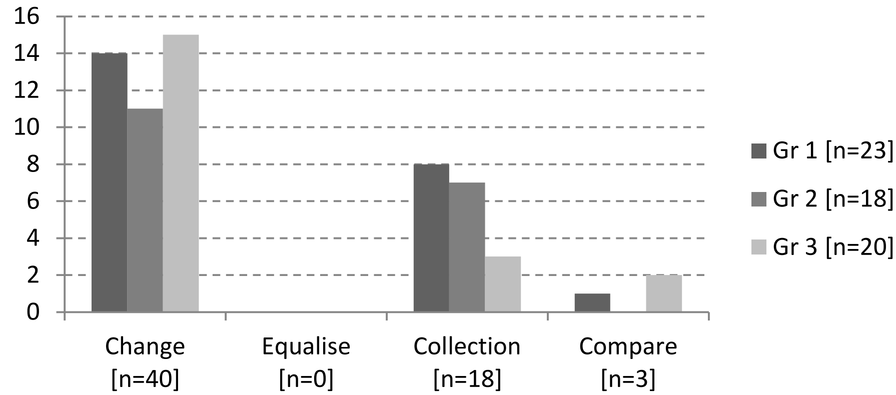

Figure 1: Distribution of word problems across four main categories, by grade

In terms of the distribution across the subcategories, result unknown problems are more prevalent than change unknown problems and there are no start unknown problems. Similarly, there are no compare type problems where the set to be acted on (i.e. the referent) is unknown.

Riley et al. (1984) reported on the results of a number of studies that determined the relative difficulty of different word problems. These studies showed that, for change type problems, children find „result unknown" problems easier to solve than „change unknown" and „start unknown" the most difficult to solve. For collection type problems, the „collection unknown" subcategory is significantly easier than „subset unknown" subcategory. For compare type problems, „referent unknown" problems are more difficult than „compared quantity unknown" or „difference unknown" problems. Therefore, the problem types that learners find more difficult to solve are the ones that are less prevalent in the workbooks. This is another reason for concern and more preferable distribution would be one in which the number of problems in the more difficult subcategories increased with an increase in grade.

CONCLUSION

The analysis of the DBE foundation phase mathematics workbook reveals a very limited number of word problems (only 61 across three grades). In particular, there are no equalise problems and only three compare type problems. This is a reason for concern in the light of the central role played by the DBE workbooks in many no fee schools. The very limited number of compare type word problems is particularly concerning due to the consistently poor performance on word problems as highlighted in the 2012, 2013 and 2014 ANA diagnostic reports.

In their analysis of the Grade 3 DBE workbooks as curriculum tools, Hoadley and Galant (2016) conclude that it is not viable to use the current workbooks as a teaching or transmission tool, primarily because of the limitations of conceptual signaling. The findings of this paper support this conclusion by showing that the frequency and distribution of the additive relation word problems in the workbooks are not suitable for them to be used as a teaching tool for additive relation word problems.

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WHAT ARE THE IMPLICATIONS OF THE NBTP QUANTITATIVE LITERACY TEST RESULTS FOR TEACHERS?

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Students intending to enter most Higher Education (HE) institutions in South Africa, write the National Benchmark Tests (NBT) to provide a measure of their readiness for HE. Numerous school-leaving learner are underprepared for HE. This article explores the following research question: Which quantitative literacy competencies that are required in Higher Education, as identified through the National Benchmark Test Project (NBTP) are not well developed by test writers and what are the implications thereof for teachers? Sixteen test item results in which students performed the poorest, were analysed (N=2348). The areas identified for teachers to focus on included data handling, percentage change and shape and space. Teachers therefore need to present learners with more challenging tasks than just to read off graphs/charts/tables by merging information of more than one graph/chart.

INTRODUCTION

Students who intend on entering Higher Education (HE) write the National Benchmark Tests Project (NBTP) tests to provide a measure of their readiness for HE. A fair number of these school-leaving learners, "are to some extent unprepared for higher education" (Prince & Firth, 2017, p1). Learners‟ competencies revealed through the test results can be of significance to HE in gauging what HE needs to focus on in interventions to support students to cope with the curriculum. The results are also relevant to the school teachers‟ community as they can also serve in guiding school teachers on what to focus on more in preparing learners for HE. This article investigates the main gaps in test writers‟ competencies identified through analysing a sample of Quantitative Literacy (QL) test results and the implications for teachers.

The research question under investigation is: Which quantitative literacy competencies that are required in HE, as identified through the NBTP, are not well developed by test writers and what are the implications of this for teachers?

This is a qualitative study where the results of test writers of a NBTP Quantitative Literacy test written in 2017, are analysed. Of the fifty items in the test, only those that fall in the bottom thirty percent in terms of the facility-value, are identified. This results in a cut-off facility value (p-value) of 0,37. This means that only those items where at least sixty-three percent of the test candidates did not choose the correct answer are considered. Data from 2 348 test writers are analysed with reference to the different mathematical and statistical ideas described in the test construct.

QUANTITATIVE LITERACY

Quantitative literacy is taught in South African schools predominantly through the subject Mathematical Literacy. This subject intends to enhance learners" ability to flourish in a "world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways" (NCS Mathematical Literacy, 2011, p 8). The five key elements of this subject are: elementary mathematical content, authentic real-life contexts familiar and unfamiliar problems, decision making and communication and the integration of content and/or skills in solving problems.

Quantitative Literacy is defined by the NBTP as "the ability to manage situations or solve problems in practice, and involves responding to quantitative (mathematical and statistical) information that may be presented verbally, graphically, in tabular or symbolic form; it requires the activation of a range of enabling knowledge, behaviours and processes and it can be observed when it is expressed in the form of a communication, in written, oral or visual mode" (Frith & Prince, 2006, p 30). The test items assess qualitative literacy competencies where the mathematical content is embedded in authentic contexts and graphs, charts, diagrams and tables were used to display the data. The test is in English (or Afrikaans) and no calculators are allowed when writing the test. The Academic and Quantitative Literacy test components are combined as one test where the QL part forms two fifths of the combined test of three hours. All the questions are multiple choice questions with four options to choose from where the distractors are carefully selected to reveal possible misconceptions.

DATA ANALYSIS

Sixteen out of the fifty items in the test were identified as items where at least sixty-three percent of the candidates chose the incorrect answers. The most prominent areas where difficulties were identified were with data handling, ratio and proportions, percentage change, working with problems where more than one operation is required and working with the surface area of a three-dimensional object. These five areas of difficulty will be used as themes in discussing the results of the data analysis.

Data handling abilities, like reading and interpreting information off graphs, are important for making sense of statistically based scenarios in everyday life and in academic disciplines. There is however evidence that this ability is not well developed by a significant number of the test candidates. To read the appropriate information off more than one given graph/table and then to make decisions pertaining to the problem posed, seems to be a daunting task. One of the items analysed tests this ability, combined with determining a percentage of a percentage. Only 19% of the test candidates managed to answer it correctly. Nearly two thirds (62%) of the test candidates chose a specific incorrect answer which shows that they did not recognise that the question actually refers to a subset of the whole, and not the whole set.

Another item that assesses the meaning of “at most” and “more than” includes reading values off a chart, but only 28% of the test candidates could answer it correctly. Candidates mostly chose the distractor that meant “not more than”. In another item that refers to a stacked bar chart, test candidates had to read off the graph to verify if statements are true or not. However only 33% chose the correct statement amongst different verbal statements. Here the candidates were confusing proportion and absolute numbers. This is a concept that needs serious interventions by high school teachers.

An item testing the ability to apply successive percentage increases over a period of time was answered poorly by candidates. In one such item, only 28% of the candidates could answer the item correctly. The option mostly opted for and chosen by 33% of the candidates, was using the same starting value for all the years and not realising that for the next year, the initial value will be different. The option where only the increase for the first year was calculated despite the longer period indicated was chosen by a quarter of the candidates. Another option chosen by 14% of the candidates was devised by using incorrect percentage increase values and using the incorrect period for the percentage change. When the concept of percentage increase was assessed using a chart only 13% of the candidates opted for the correct answer, while 49% of the candidates chose the absolute increase instead. Another misconception was evident when learners had to distinguish between the highest percentage increase and the highest measured value evident in a chart. In an item that tested this, 49% of test candidates chose the greatest absolute increase and not the percentage increase, while 32% of the candidates opted for the highest values represented on the chart.

Determining surface area of a given three-dimensional object was another ability also not well developed by test candidates. In an item testing this ability, 49% of the candidates selected the answer that was the calculated volume of the shape, not the area. Only a quarter of the candidates could answer it correctly. Another item on determining the area of a regular shape included conversion of the unit of measurement. One distractor chosen by 33% of the candidates represented one side and not the area. Another distractor where the length was converted and not the area, was selected by 25% of the candidates. Another distractor where conversion did not take place, was selected by 20% of the candidates. This clearly shows the test candidates‟ inability to work with situations involving calculating area and converting units.

Ratio and proportion questions were also not answered well. In one item, where the data was given in table form and the candidates had to identify the ratio of two different numbers in the table, only 34% of the candidates could answer it correctly. Either the parts were in the incorrect order (21%) or the total was written in terms of one part (44%). Another item on ratio and proportions required test candidates to construct a formula which was expressed the ratio verbally. However, statements of ratios in words could not be correctly translated. Only 35% of the candidates could answer it correctly. 51% of the candidates did not show insight in converting the verbal ratio into the form $\frac{a}{b} = \frac{c}{d}$. The results of another item on ratio and proportions reveal that test candidates fail to recognise the different options of representing ratios. The representation a : b and $\frac{a}{b}$ could be recognised but not the other valid representations.

Comparing different sub-sections in terms of the total and then presenting it as a fraction using a line graph, is a concept that was also a challenge to a significant portion of the test candidates. In an item a fraction had to be determined by reading it off a multiple line graph. Only 32% of the candidates could answer it correctly. The option closest to the answer was selected by 17% of the candidates while other options further away from the correct answer respectively were selected by 27% and 22% of the candidates.

DISCUSSION

The NBTP QL test results clearly identify certain topics in the high school curriculum that teachers need to be aware of in which learners do not show well developed abilities. The exposure to authentic contexts in which problems are set is important in the development of these abilities. Learners need to experience not just authentic, but also varied contexts to master problem solving skills after the basic concept is dealt with in class.

Data handling is crucial for learners‟ skill to develop in a world "characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways" (CAPS, 2011). Teachers therefore need to present learners with more challenging tasks than just reading values off graphs/charts/tables. The reading of data off graphical information should be combined with identifying the "whole" as well as subsections of the "whole".

Learners need to be supported in activities requiring higher order thinking where they are exposed to settings pertaining to data and real-life contexts. One step questions should be followed up with multi-stepped questions.

Percentage change is a concept which seems quite challenging to learners to understand. Therefore, more effort and time needs to be invested in learners understanding the concept. Concept development needs to include the use of graphs as well as data tables to strengthen a deeper understanding of percentage change. It might be that teachers underestimate the difficulty level of understanding percentage change.

Calculating area should be combined with converting units as well as distinguishing the calculation of area from that of volume. The calculation of area and volume should be dealt with in conjunction with visuals of the objects and identifying the surface area thereof. This can support concept formation in such a way that learners can also determine area and volume of irregular objects.

CONCLUSION

Teachers should identify learners" gaps in knowledge and abilities and try to adapt their way of teaching to optimally support learners" concept development as opposed to recipes and the use and application of formulae. This can support learners in becoming self-managing individuals in a world where mathematical content is imbedded into real-life context.

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ENHANCING TEACHING OF MATHEMATICAL LINEAR PROGRAMMING USING THE REALISTIC MATHEMATICS EDUCATION APPROACH AT TVET COLLEGES

Khehla Patrick Mofokeng

The purpose of this study is to enhance the teaching of mathematical linear programming using the Realistic Mathematics Education (RME) approach at Technical and Vocational Education and Training (TVET) Colleges. Various studies indicate that there are problems with the teaching and learning of mathematical linear programming which leads to the poor performance in mathematics (Chinamasa, Nhamburo and Sithole, 2014:57; & Olubukola, 2015:846). The (RME) approach address the challenges of teaching mathematical linear programming, as it allows lecturers to teach MLP starting from informal to formal (linking content with real-life stories) in order for students to make sense of teaching and learning of MLP. This study uses the RME approach to decolonise mathematical linear programming by integrating real-life stories into teaching process. The study adopts bricolage as theoretical framework to strengthen the teaching and learning of mathematical linear programming as it allows multiple strategies and creativity to solve apparent problems by resources at hand. In addition, this study uses Participatory Action Research (PAR) to generate data through the interactions between co-researchers' (myself included) during the meetings, workshops, discussions and presentations.

INTRODUCTION AND BACKGROUND

This study aims to enhance teaching and learning of mathematical linear programming at Technical and Vocational Education and Training colleges. The diagnostic report of the Department of Basic Education (DBE, 2013: 127) indicated that students did not perform well in mathematical linear programming. In addition, markers in the Department of Higher Education Training (DHET, 2013: 31) report confirmed that students have a serious challenge with mathematical linear programming. Therefore, these reports prove that students" poor performance in mathematics and the high rate of retention and dropout at TVET colleges are the results of difficult subjects and difficult topics that students do not understand well (DHET, 2013: 33-34). Moreover, various studies which have been conducted by the researchers agrees that mathematical linear programming is still challenging for students to do well in different countries such as Slovakia, Zimbabwe and South Africa (Olubukola, 2015: 846; Chinamasa, Nhamburo, Sithole, 2014: 57; DBE, 2013; & DHET, 2016). Mathematical linear programming refers to mathematical concept that consists of a set of variables and constraints that is expressed in objective functions/ linear equations (Daniel and Solomon, 2011: 178). It is used to find best solution to the present problem of particular project and also applied in manufacturing, public health, constructions, personnel planning, and investment management especially in maximising profit/ lowering costs (Nkambule, 2009: 07).

Molnar, (2016; 2-12); Chinamasa et al, (2014: 57) & Kar (2016: 652) says most of students struggles to do the following when solving mathematical linear programming: to write a set of mathematical variables, to formulate linear functions both from a case studies; to draw and interpret graphs. In addition, Okello, (2010: 212-215) said most of students lack of interest to do mathematics, have wrong perception on mathematics, do not understand mathematical linear programming terminologies, and do not ask questions during lessons or even practice mathematics at home while highlighting that most of lecturers only use lecture method to teach mathematics and also fails to integrate real life stories in teaching process. To enhance teaching and learning of mathematical linear programming most of researchers (Kar, 2016; Molnar, 2016; Nakhanu, 2015; and Van den Heuvel-Panhuizen & Drijvers, 2014) showed that different strategies „computer software, problem posing, problem-based-learning and Realistic Mathematics Education" approaches can be used to teach mathematics. There could be challenges in some strategies e.g. computer software needs a lot of money for programme to available and problem posing could be challenging to students if lecturers" fails to associate teaching with real life stories. However, Realistic Mathematics Education approach it integrates background knowledge of students, real life stories into teaching process in order for students to make sense of their learning (Dickinson and Hough, 2012: 1-6). Therefore, this study adopts RME to teach mathematical linear programming.

The approach requires classroom environment that is welcoming and well better for all students in order for them to interact without any hindrance. To teach mathematical linear programming using RME approach lecturers and students must be creative in the classroom as bricoleurs (Mahlomaholo, 2013). According to Van den Heuvel-Panhuizen et.al. (2014: 521-524) says RME needs curriculum; work schedule; lesson plan; reformed textbooks; class-activities. Additionally, co-researchers should be capacitated on the six core principles guiding RME approach such as activity, reality, intertwinement, interactivity and guidance well in order to implement it well. However, there are many threats to disturb RME approach not to work. Papier, (2009: 38-41) said shortage of well-trained lecturers, inadequate of teaching facilities, English as teaching language, class overcrowding, work-pace, unlimited of time, unavailability of variety of quality textbooks and study-guides affects teaching and learning negatively. But for RME approach to work co-researchers will be requested to come with NCV level 3s and grade 11s mathematics textbooks and also be allowed to use their languages to participate.

The initiatives were taken in the past to curb alarming poor performance in mathematics and such interventions did enhanced teaching and learning of mathematics. Computer software were used in Slovakia and it assisted students to gain deeper knowledge on linear inequalities while problem- posing allowed students to associate mathematical concepts with daily life situation in Turkey and Kenya. Lastly, RME approach was used to teach mathematics in various countries Holland (1971), United State of America (1996), United Kingdom (2003), Indonesia (2016) and has played significant role in assisting students to make sense of their leaning which led to better performance in mathematics (Dickinson et.al. (2012: 01).

The previous researches and yearly reports confirmed that students fail mathematical linear programming. Most of students are experiencing the following challenges (Molnar, 2016: 2-12): to set variables and formulate problem into objective function from a case-study, to draw and interpret graph.

The study seeks to use bricolage as theoretical framework for strengthening the teaching of mathematical linear programming. Loarne, (2010: 2) explains bricolage as a process creating something new out of the resource available for purpose of achieving new goals. Bricolage encourages co-researchers to contribute their knowledge and resources that will be used at any time to strengthen the teaching of mathematical linear programming. Additionally, the bricoleures will address recent issue in education by decolonising education using RME strategy to teach mathematics in order for student to make sense of the learning (Mahlomaholo, 2013). Therefore, I adopt bricolage to this study because it encourages lecturers, students to use multiple methods to solve mathematical linear programming problems/ challenges.

How can RME strategy be used to teach mathematical linear programming?

Research aim:

To enhance the teaching of mathematical linear programming by using RME strategy.

Objectives:

- To identify the challenges to teach mathematical linear programing.
- To use RME strategy to address challenges to teach mathematical linear programming.

- To identify conducive environment to teach mathematical linear programming using RME strategy.
- To identify threats that may hamper RME strategy to work and investigate on how to prevent them.
- To find the indicators of success for using RME strategy to teach mathematical linear programing.

Methodology

This study will use Participatory Action Research (PAR) as the tool to enhance teaching and learning of mathematical linear programming. Denzin and Lincon (2007: 277) explains Participatory Action Research as a social process where group of people join together to share their experiences to solve the problems at hand because all people have valuable knowledge from their background environment and culture. Secondly, he said PAR involves a number of stages of implementation, namely planning, action, observation and reflection that follow each other spirally. The study regards PAR as significant tool to be used diagnose apparent problem to teach mathematical linear programming in order to find solutions through participation and interaction between co-researchers (Moloi, 2013: 480).

The study regards co-researchers as people who can speak, who knows challenges better (teaching mathematical linear programming) and who can provide solutions with regard to their local issues if they are given platform. Data will be generated through meetings, workshops, team discussions, and lesson presentations. In addition, audio record, videos, cameras will be used to capture proceedings of the meetings with co-researchers because a scriber cannot write very word communicated. Therefore, the co-researchers as the team will draw up an action plan on activities to be efficient to generate data. Additionally, this study uses participatory action research to allow voice of people to be heard through interactions in the class.

The study will use a group of 35 National Certificate of Vocational (NCV) students of particular class (Level 3), three mathematics lecturers, two Senior lecturers, Campus (HoD)/ Campus manager. The selected people are chosen because they have different background knowledge and experience, so they will help us to find challenges to teach mathematical linear programming and also assist the study in finding solutions (Cerecer, Cahill and Braley, (2013: 218); & Moloi, (2013: 480)). Lastly, the study sessions aimed to be held at TVET campus during mathematics class but it is still open for discussion, so co-researchers will decide days, place and time that they will be suitable for them.

This study will use Critical Discourse Analysis to analyse and interpret generated data in order to gain deeper understanding of text and talk on tape records and other used records. According to Bloor and Bloor (2007: 2) explains CDA as cross discipline that comprises the analysis of text and talk in almost all field of humanities and social sciences. Therefore, this study will allow co-researchers to use the languages they feel free with for interaction purpose.

There is need to conduct this research on mathematical linear programming because yearly reports and various research have showed that mathematical linear programming is challenging for students (DBE, 2013: 127; & DHET markers reports (2016:31). Therefore, this study anticipates strengthening teaching and learning of mathematical linear programming using RME approach, so TVET college students, lecturers, TVET college, community, and DHET will benefit from this study.

The study will seek letter of permission from DHET to conduct the research at TVET college premises. Secondly, the Faculty of Education at UFS will issue ethical clearance which confirms approval for this study to be conducted. Thirdly, all people who are interested to be part of this study will be asked to fill and sign the consent forms. However, people under age of 18 will not be allowed sign these forms but their parents will sign for them. The consent forms will be translated in English, Sesotho, IsiZulu and other languages people understand. Lastly, co-researchers will be informed that they may join/ leave/ re-join the study at any time, but most importantly they will be assured anonymity with information they supplied/ their personal details.

RESEARCH STRUCTURE/LAYOUT

- Chapter 1: Introduction, background
- Chapter 2: Theoretical Framework and literature review
- Chapter 3: Research Design and Methodology
- Chapter 4: Data analysis, interpretation and reporting
- Chapter 5: Conclusion and recommendations

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HOW TO EFFECTIVELY APPLY INSTRUCTIONAL LEADERSHIP AS A STRATEGY TO LEAD MATHEMATICS TEACHERS

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Leadership methodology plays an important role in the education system. The South African Department of Basic Education has made numerous changes to the education system of the country with the objective of improving learner results. However, despite all the modifications to the South African education system, learners still battle to achieve the desire results. This paper argues that the problem does not lie with the education system, but rather with the leadership style adopted by the schools. This paper proposes that Instructional Leadership is the most effective and efficient leadership strategy that can be adopted by those in management in order to improve mathematical performance of learners. Instructional Leadership places emphasis on the fundamental roles of each and every leader in the school environment. In order to be centers of academic excellence for mathematics education, schools, through their leadership, should adopt the principles and prescripts of Instructional Leadership which place emphasis on leader roles and achieving the basics. This paper aims to highlight the significance of Instructional Leadership as a leadership methodology that helps accelerate the achievements of excellent mathematics results. The paper provides an in-depth analysis of Instructional Leadership as a leadership methodology that could advance and improve mathematics teaching. Instructional Leadership is discussed within the context of mathematics leadership, as assumed by Heads of Mathematics Departments.

INTRODUCTION

The core responsibility of every teacher is to advance his/her student‟s comprehension of mathematics and subsequently produce desired results. This responsibility has produced an additional burden, especially for teachers of mathematics who are faced with the daunting task of simplifying a subject that is generally perceived by society to be almost impossible to master. This observation was reinforced recently at the Johannesburg District Information Forum where a group of teacher experts deliberated on promotion requirements for grade learners, and one presenter posed a provocative question to members of the School Management Team, “Whose child must fail?” The question makes additional statements that challenge educators: Educators, as members of their communities, should not only be skilled, but must be passionate about their work and must strive to do justice to the children and their parents.

Ethical educators must thrive to create conducive atmospheres for learners in order for them to maximize their potential. Therefore, the answer to the question should be, “No child should fail.”

Unfortunately, the current status is that mathematics results contribute to the overall higher failure rate in our schools. Yet, the major cause of the failure rate is not always the subject, mathematics per se, but rather faulty leadership skills and misguided leadership methodologies adopted by those in leadership.

The objective of my presentation is to:

- Contextualise the South African mathematics problem
- Define Instructional Leadership
- Identify Instructional Leaders
- Highlight the roles of Head of Department (HOD) as an Instructional Leader
- Make recommendations and conclusion

THE SOUTH AFRICAN MATHEMATICS PROBLEM

One of the biggest challenges in South Africa‟s schooling system since the dawn of democracy remains mathematics teaching. Spaull (2013: 42) confirms in his report that learners‟ learning deficits that are acquired at primary school level discourage them from following streams like mathematics and sciences, since they had not grasped basic fundamentals of mathematics. Various strategies have been introduced by government in order to tackle the dilemma of mathematics teaching, ranging from dividing mathematics into two subjects, mathematics and mathematics literacy, to lowering the minimum mark required to progress in the senior phase – grades 7, 8 and 9, to 20%, with the argument that those learners who achieve less than 20% would not pursue careers in mathematics in any case (Department of Education, 2011b).

Cohen et al, (2008: 42) argue that from a myriad of reasons behind poor performance of learners in mathematics, teacher teaching methodology and leadership have the most direct impact on the achievement of envisioned results.

This sentiment is echoed by Chad et al, (2012: 39) who assert that the best performing countries in terms of mathematics, such as Japan and China, make great investments in teacher training, which encapsulates teaching methodology. Spaull (2013: 58) has alluded that the crisis in South Africa Education system is in relation to not having clear systematic implementation plans that will improve the quality of learning and teaching. This paper proposes that Instructional Leadership is the most effective and efficient strategy that can be used to improve mathematical performance of learners, if properly adopted and implemented.

WHAT IS INSTRUCTIONAL LEADERSHIP?

Darling Hammond et al, (2010: 66) articulate that Instructional Leadership involves working with classroom teachers to improve instruction; providing resources and professional development aimed at improvements in instructional capacity; coordination of curriculum; instruction and assessment; regular monitoring of learners and teacher performance; cultivation of the learning culture; with special focus on improvement in teaching and learning (Darling Hammond et al, 2010: 66).

Additionally, the word "instructional" as explained by Cambridge English Dictionary means to teach someone how to do something. "Leadership" means to guide, to direct, authority or control, managing a group of people or organization. The school is a complex entity that encompasses different components such as learners, teachers, school management team, support staff, resources and buildings. All these components need to be managed and cannot be managed by one person, the principal Muhammad, A. (2015: 45). Thus the principal requires a team of effective, efficient and professional educators who will manage the school environment together with its dynamics (Prinsloo, 2008: 38).

WHO ARE INSTRUCTIONAL LEADERS?

As stipulated in the Personnel Administrative Measures (PAM) document of 2016 (Annexure A.5, pp A-27 - ppA-29), each and every member of the School Management Team (SMT) has core duties and responsibilities of their respective jobs. In essence, a school cannot be managed by one person as the African proverb says, "It takes a village to raise a child." All SMT members should be instructional leaders; Head of Department (HOD) leads teachers of a particular learning area such as Mathematics, Sciences, Humanities and Languages, and Deputy Principals lead different Phases.

THE ROLES OF A HEAD OF DEPARTMENT (HOD) AS AN INSTRUCTIONAL LEADER

Working with classroom teachers to improve instruction

As stipulated in the PAM document (Annexure A.5, pp A-27 - ppA-29), the core responsibilities of the Head of Department (HOD) at school are to facilitate teaching, extra & co –curricular activities, administer personnel, general /administrative and communication tasks. The document further states that it is the responsibility of the HOD to interact with the teachers as a curriculum specialist and to improve instruction. As an instructional leader, the HOD should lead from the front, which means that he must always conduct research and be passionate about mathematics and be up to date with the latest developments in the subject (van der Berg et al, 2011: 37).

The teaching responsibilities of the HOD would depend on the workload or needs of the school. The HOD must be in charge of a particular learning area such as mathematics, provide guidance on the latest ideas and approaches to the subject area, and identify modern teaching methodologies, techniques and relevant teaching aids (Chad, 2012: 62). The HOD must also be responsible for coaching and monitoring of new incumbents. Additionally, on a strategic level, the HOD as an instructional leader has to ensure that the school affiliates with professional bodies such as AMESA, and must ensure that teachers in his department participate in such bodies as part of staff development. In this way, teachers will be well positioned to be capacitated with skills and knowledge in their learning area, which will be to the benefit of the learners, and will yield desired results in the long run (Reddy et al, 2015: 59).

Providing resources and professional development aimed at improvements in instructional capacity

For effective learning and teaching to occur resources must be available at school. According to Schollar (2008: 49) learners fail to progress from Foundations Phase in basic computing skills (Grade 1-3) to Intermediate Phase (Grade 4-6) and then to Senior Phase (Grade 7-9) because of predominance of concrete over abstract methods of calculating. There must be sufficient mathematics textbooks for all learners, teachers‟ guides and relevant mathematical equipment for the mathematics department to create a conducive learning environment for learners to be stimulated to be able to tackle higher order questions as per Bloom‟s Taxonomy (Chad, 2012: 48). All schools have Learner Teacher Support Material (LTSM) committees that are responsible for procurement of resources for the schools. HODs and deputy-principals are generally part of the committee by virtue of being members of management. Heirdsfeild et al, (2014: 42) state that if teachers work together and interact with content that needs to be taught, they become masters as they share good practices. As an HOD, it is imperative to develop teachers within the mathematics department at your school. This can be achieved by using Professional Learning Committees (PLC) effectively. Furthermore, schools that are within close vicinity can form a cluster and share information, lesson plans and best practices. HODs can be drivers of such initiatives which will demonstrate Instructional Leadership.

Coordination of curriculum, instruction and assessment

As instructional leaders, HODs and managers are responsible for the successful implementation of the mathematics curriculum in the school (Muhammad, 2015: 19). A leader does not wait to be invited but invite him/herself. Instructional leaders are always prepared and have systems in place to check to what extent the content of the mathematics curriculum has been covered (Heirdsfeild, 2010: 33).

The HOD must also assess if the Annual Teaching Plan (ATP) is being followed as scheduled and if there are any deviations and reasons for such deviations must be submitted in writing. It is the responsibility of the HOD to guide teachers on assessment tasks and to ensure that those tasks adhere to Department of Basic Education (DBE) regulations and comply with CAPS prescripts. The HOD should ensure that assessment tasks should be valid, reliable, fair, balanced and comprehensive.

Regular monitoring of learners and teachers' performance

Leadership also encompasses monitoring, so as instructional leaders, HODs must monitor learners‟ books in order to establish content coverage within the mathematics department. The Department of Education‟s School Based Assessment (SBA) provides a true reflection of learners‟ and teachers‟ performance. Moderation of SBA and using moderation tools that indicate level of competence and class average will indicate to the HOD the performance of learners and teachers in mathematics. Such moderation will assist the teacher and HOD to come up with intervention strategies that will improve attainment within the mathematics department (Reddy et al, 2015: 59). This is important as it will assist with remediation before it is too late. In this way, learners will be familiar with the process and teachers will be able to re-teach using different strategies.

Cultivation of the culture of learning and teaching in schools

As an instructional leader who is responsible for the management of the day to day running of the mathematics department, it is imperative that the culture of learning and teaching is infused within your department (Cohen et al, 2008: 35). HODs should motivate their respective teachers within the departments they head to be able to create a conducive learning environment in classrooms which will allow learners to have a positive feeling about learning. According to van der Berg et al, (2011: 37) the culture of the classroom depends on many factors such as classroom arrangements, availability of resources, but of utmost importance, is the efficient use as well as the safeguarding of those resources. Instructional leaders have mentoring and coaching sessions with teachers within the different departments and they strategise on how to create a culture of learning and teaching.

In an effort to cultivate the culture of learning and teaching, the Gauteng Department of Education adopted a vision of "Each Child Matters." This vision provides the mathematics department with a clear direction and an instruction that the department must:

- Set high expectations
- Encourage teachers and learners to have positive interactions with each other
- Teachers to give learners a voice during class interaction
- Make classroom a safe place to be in
- Model how learners can learn
- Give regular feedback
- Not only celebrate percentages but accomplishments too.

Video clip on Culture of Learning and Teaching (Muhammad, 2015: http://www.teachertube.com/video/transforming-school-culture-250291)

RECOMMENDATIONS FOR HEADS OF MATHEMATICS DEPARTMENTS

Flowing from the core principles of Instructional Leadership, the following recommendations are made for Heads of Department (HOD) for Mathematics:

- Introduction of compulsory mathematics tutoring: William et al, (2013) reinforce that peer tutoring with purpose assists with improving academic achievement in order to effect mathematics teaching reinforcement. The mathematics HOD should introduce compulsory mathematics tutoring system that should take place after school. According to Prinsloo (2008: 29) having a tutoring programme with correct tutors in place will assist with improvement of mathematics achievement. The tutoring system should be part of the school timetable. Furthermore, tutoring should be done by either current of former learners who reside in the surrounded community of the school. The approach of the learner tutor removes the psychological barrier learners might have in their fears for both the teacher and the subject.

- The mathematics Head of Department must introduce compulsory monthly workshops that aim to create platforms for exchange of good practices from either fellow mathematics teachers or invited mathematics experts.

- The HOD should identify best performing schools in mathematics in the region, and establish collaborations and partnerships with the mathematics department of those schools, with the objective of extracting best practices from them.

- The HOD should invite excellent mathematics practitioners to present and share their best practices in mathematics learning.
- The HOD should introduce the reward system that celebrates top and most improved mathematics learners.

CONCLUSION

Since the dawn of democracy, in an attempt to improve learner performance and school results, the South African government has been making numerous amendments to the education system, accompanied by large financial investments in the education department. The government has made changes to the curriculum, minimum pass requirements as well as learning areas, however, there has not been any significant improvement in the results. This paper has argued that the most important change that the South African government needs to introduce is the shift towards and emphasis on Instructional Leadership. As clearly articulated in this paper, Instructional Leadership takes those in leadership back to their basic responsibilities. Instructional Leadership provides clear and basic guidelines and responsibilities for each and every member of leadership. Therefore, with the adoption and successful implementation of Instructional Leadership, the South African Education System is set to yield the desired results and will meet its targets, and be transformed into one of the best in the world.

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THE TEACHING AND LEARNING OF MATHEMATICAL CONTENT: COMMUNITY CULTURAL WEALTH APPROACH

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The paper aims at enhancing the teaching and learning of mathematical content using the community cultural wealth theory. This theory amplifies the notion that mathematics is a humanistic subject, that is, where all learners have access in mathematical knowledge. The theory views community members as experts in the teaching and learning of mathematical knowledge. The paper argues that there are no deficiencies in the marginalised knowledge of the excluded people. As the results, the researcher tapped into the marginalised knowledge of subaltern communities to mathematical content, using participatory action research (PAR) in generating data. Hence, the involvement of community members (parents, traditional leaders), education experts (teachers, mathematics subject advisors, lecturers from institution of higher learning) and learners themselves. The generated data were analysed using Van Dijk's critical discourse analysis (CDA). CDA enabled the study to acquire deeper meanings of the text.

INTRODUCTION

The paper argues that Community Cultural wealth approach is key in the teaching and learning of mathematical content. This is confirmed by Samuelson and Litzler (2016) that Community cultural wealth offers an assets-based approach to understanding the huge cultural knowledge that learners bring in a mathematics class, and remained untapped by mathematics practitioners. The experiential knowledge of learners from rural communities is critical in understanding mathematical content. community cultural wealth will help to identify the cultural resources that learners from subaltern families and communities bring to mathematics classroom to learn mathematics meaningfully (Stapleton, 2016; Straubhaar, 2013).

BACKGROUND AND RELATED LITERATURE

The Department of Basic Education Report (2009) posits that many of the mistakes made by learners in answering the mathematics assessment tasks had their origins in poor understanding of the basics and foundational competencies taught in the earlier grades. The teaching of mathematical content need to encourage the deep conceptual understanding of concepts. Thus, this paper will focus on enhancing the teaching and learning of mathematical content using cultural games, by paying attention to certain topics in which learners in lower grades do not perform well. Cultural game is regarded as the cultural wealth that learners bring in a mathematics classroom. Further, this paper tends to contribute towards learning and teaching mathematical content in a creative way.

Gaigher, Rogan and Braun (2006) point out that most learners perform badly because of teacher-dominated approaches, and the learners expected to remain passive recipients of rote learning. Troutman and Lichtenberg (2003) state that teachers need to provide learners with stimulating problem-solving activities. This paper, therefore, uses diketo as a stimulating activity in the teaching and learning of mathematics. Mosimege (2000, 2017) clarifies misconceptions about cultural games, namely that they are usually perceived from the narrow perspective of play, enjoyment and recreation, whilst there is actually more to them than just the three aesthetic aspects. Indigenous games can reveal mathematical concepts associated with them. According to Van De Walle, Karp Bay-Williams (2010) and Leonard (2008) in the traditional modes of teaching, learners do not learn content with deep understanding and often forget what they have learned. Thompson (2008) suggests that teachers should capitalise on the background of learners for performance to be enhanced. Children meet mathematical concepts every day and operate in rich mathematical contexts even before they set their eyes on a mathematics worksheet.

The researcher and the research participants argue that the teaching of mathematical content is abstracted and treated as if knowledge of it ends only with memorising mathematical formulae. The community cultural wealth employs capitals such as familial, aspirational, navigational, linguistic, resilience (Samuelson and Litzler, 2016; Stapleton, 2016; Straubhaar, 2013; Yosso, 2005), which can be used to access the hidden mathematical content. Troutman et al. (2003) support the latter statements that teachers need to strive to build a foundation and master important teaching techniques related to problem solving. Van De Walle et al (2010) points out that mathematical content knowledge grounded in an experience familiar to learners supports the development of advanced mathematical concepts and provides them with access to meaningful mathematical reasoning, thus they are able to learn it successfully.

On the other hand, Hirsh (2010) argues that if mathematics teachers continue to teach what they know and ask learners to memorise and regurgitate content it is impossible to expect any advancement to be made in mathematical content knowledge. To a large extent the teaching of mathematics content lacks the element of relevancy to real life; hence, learners do mathematics for the sake of passing tests or examinations, and with little conceptual understanding. They are not taught the skills of deriving formulae and functionalising knowledge derived from them. Koellner, Jacobs, Borko, Schneider, Pittman, Eiteljorg, Bunning, Frykholm (2007) points out that mathematical content should not be merely taught as a set of procedural competencies but rather mathematics teachers had to help learners gain adequate conceptual knowledge along with a flexible understanding of procedures to become competent and efficient problem-solvers. Learners are thus limited in terms of creativity and self-discovery as a result of this way of teaching mathematical content. Thompson (2008) argues that children are likely to be creative when they use ideas and experiences, and make new connections through play.

THEORETICAL FRAMEWORK

The paper is grounded by community cultural wealth theory, focusing on the wealth of knowledge which the marginalised groups possess. Such knowledge is key to the teaching and learning of mathematical content. Theoretical framework posits that teaching and learning of mathematical content is created by the marginalised groups out of their everyday lives experiences, rather than being taught in a formal setting.

In addition, Lynn (2004) and Yossi (2002, 2005) argue that community cultural wealth concentrates on and learns from the range of cultural knowledge, skills, abilities and contacts held by subaltern groups that often go unrecognised and misunderstood. In this paper, the use of cultural games in teaching mathematical content and skills is a way of recognising and acknowledging the cultural practices of various communities.

Yosso (2005) argues that community cultural wealth theory has various forms of capital, such as aspirational, navigational, social, linguistic, familial and resistant. Aspirational capital relates to the motivation shows playing their cultural game. Whereas navigational capital looks at how learners manoeuvres to win in any game. Also familial and linguistically demonstrate how learners‟ networks (parents, peers, learners) are essential in learning language easily. These draw on knowledge of learners from homes and communities being taken into the classroom environment. The researcher supports Yosso‟s theory of community cultural wealth and Van Oers‟s cultural-historical theory, in that using cultural games to teach mathematics content and skills is a way of bringing the immediate environment and experiences of the child to the classroom. Van Oers (2010) points out that children learn mathematical content optimally when their learning is deep-rooted in playful activities.

Whilst it is evident that the teaching and learning of mathematical content should be viewed from a humanistic point of view (Barker, 2012; Bush, 2005; Vilela, 2010), through the lived experiences of marginalised groups there are many mathematical concepts that are formulated. Mathematical content meanings are not fixed or predetermined, and meanings are not indifferent to linguistic practices (Lynn, 2004; Vilela, 2010; Yosso, 2005). The link between mathematical content and the cultural practices (such as play of cultural games) helps learners to see and appreciate the relevance of mathematics skills in their day-to-day activities (Chikodzi & Nyota, 2010).

METHODOLOGY AND DESIGN

The study utilised participatory action research (PAR), which recognises community members as experts and is empowering for communities who are enabled to find their own solutions to local issues (Moana, 2010). In the context of this paper, the researcher and participants were empowered in using cultural games to solve problems and identify mathematical concepts embedded in them. The marginalised capital was explored to understand mathematical content by using cultural games, particularly indigenous ones. As Yosso (2005) argues, there is much cultural capital in the communities which is not being adequately utilised.

The researcher assembled a team of grade 9 learners in one school located in the rural area of the Free State Province, one deputy principal, one head of department (HOD), three grade 9 Mathematics teachers, two life orientation teachers, two district officials from the Department of Basic Education (DBE), one in the sports section and two Mathematics subject advisors, ten parents with knowledge of various cultural games and two members of the royal family who were custodians of cultural games, and a lecturer in the school of Mathematics, Science and Technology Education from the university.

Activities performed in class

The learners were given activities on diketo as the cultural game, which that they familiar with. The activities expected them to extract mathematical content embedded within this game.

Class Activity 1

Let us consider the play of coordination game (*diketo/Izingedo*), let us focus on round one only (*seng one*). Suppose that three groups are playing with six, eight, ten small stones respectively and one big round stone(ghoen). Each group has to record all their observations with regard to:

a) The number of times you throw the ghoen up to complete round one successfully,

b) Number of times you scoop the stones out of the hole,

c) The number of times you push back the stones in the hole.

d) Finally, what general deductions can you make, and

e) Describe and mention any mathematical content recognized during this process of playing coordination game.

It must be stated that there were groups that mentioned that they use" pups" as they play diketo, while others do not use "pups". Pups is done at the end of the round, where the ghoen is thrown up and touch the hole with empty-hand.

In nutshell, during feedback sessions, one group led by Tatso (group leader), presented their solutions as follows:

Tatso: *good day, everybody, we wanted to share our solution as follows. Take note that we used "pups" in our play. The following scenarios were observed as we interacted in our group.*

Throwing the ghoen up	**No. of stones scooped out of the hole**		**No.of stones placed into the hole**
1st throw	6	**2nd throw**	5
3rd throw	5	**4th throw**	4
5th throw	4	**6th throw**	3
7th throw	3	**8th throw**	2
9th throw	2	**10th throw**	1
11th throw	1	**12th throw**	0(with pups)

Table 1: Tatso's group contribution

:as shown in the table 1 above, we started playing with 6 stones, we got 12 throws of the ghoen in order to complete round one successfully, and for 8 stones used in play, we arrived at 16 throws of the ghoen, and lastly for 10 stones used, there were 20 throws observed. Let me give over to Nku to complete other questions

Nku: with regard to question (b) and (c), column 2 shows stones scooped out of the hole, likewise, column 4 gave us stones pushed back into hole as per a particular throw. we further observed that there is a relationship between the stones and the number of throws of the ghoen. Then, the relationship between stones and throws can be expressed as $f(6) = \ 2\, times\ the\ number\ of\ stones\ = 12$, $f(8) = \ 2 \times 8 = 16$ and $(10) = 2 \times 10 = 20$. Generally, we were able to realise that the relationships between the stones and the throws of the ghoen can be expressed as: $f(x) = 2x$ (where x denotes number of stones used in the play).

On the other hand, groups that play without pups, they presented their work as follows:

Nti (the group scriber): thanks Tatso"s group, we are excited with your good presentation. Our contribution is thus,

Throwing the ghoen up	**No. of stones scooped out of the hole**	Throwing the ghoen up	**No.of stones placed into the hole**
1st throw	$6 = 6 - \frac{(1-1)}{2} = 6 - \frac{0}{2} = 6$	**2nd throw**	$5 = 6 - \frac{2}{2} = 5$
3rd throw	$5 = 6 - \frac{(3-1)}{2} = 6 - \frac{2}{2} = 5$	**4th throw**	$4 = 6 - \frac{(4)}{2} = 6 - 2 = 4$
5th throw	$4 = 6 - \frac{(5-1)}{2} = 6 - \frac{4}{2} = 4$	**6th throw**	$3 = 6 - \frac{(6)}{2} = 6 - 3 = 3$
7th throw	$3 = 6 - \frac{(7-1)}{2} = 6 - \frac{6}{2} = 3$	**8th throw**	$2 = 6 - \frac{(8)}{2} = 6 - 4 = 2$
9th throw	$2 = 6 - \frac{(9-1)}{2} = 6 - \frac{8}{2} = 2$	**10th throw**	$1 = 6 - \frac{(10)}{2} = 6 - 5 = 1$
11th throw	$1 = 6 - \frac{(11-1)}{2} = 6 - \frac{10}{2} = 1$ (without pups)	**12th throw**	
the throw	$f(x) = -\frac{x}{2} + \frac{13}{2}$		$f(x) = -\frac{x}{2} + 6$

Table 2: Nti's group workings

> :we came up with 11 throws of the ghoen for 6 stones used in the play and, 15 throws of the ghoen for 8 stones played with, and also 19 throws of the ghoen for the 10 stones used as we play. Like, the previous group, the answers for (b) and (c) are shown in column 2 and 4 respectively. Furthermore, the relationship between the stones and throws of the ghoen can be demonstrated as follows: $f(6) = 2$ times the number of stones less one $= 12 - 1 = 11$, $f(8) = 2 \times 8 - 1 = 16 - 1 = 15$. By and large, we agreed that the relationships between the stones and the throws of the ghoen can be expressed as $f(x) = 2x - 1$ (where x denotes the number of stones used).

During the learner interaction and the teachers, learners employ capitals such as linguistic, used language to visualise patterns and develop general conclusions. Navigational wealth was shown as they made trial and error to see the sequences of occurrence in order to get general conclusions. (Samuelsona and Litzler, 2016; Stapleton, 2016).

ANALYSIS OF THE DATA

The researcher team used Van Dijk‟s critical discourse analysis (CDA) in analysing and interpreting data, to get deeper meaning of the spoken words of the research participants. The research team used the CDA because it is compatible with the community cultural wealth, as it allows for various ways of arriving at the truth (Wodak & Meyer, 2009). CDA is a type of discourse analytical research that mainly examines the way social power exploitation, dominance, and inequality are sanctioned, reproduced, and opposed by text and talk in the social and political context. (Hakimeh Saghaye-Biria, 2012; Van Dijk, 2003; 2009).

The following is an example of a text, from generated data above:

> "…. we started playing with 6 stones, we got 12 throws of the ghoen in order to complete round one successfully, and for 8 stones used in play, we arrived at 16 throws of the ghoen, and lastly for 10 stones used, there were 20 throws…".

The above text was analysed and interpreted so as to get the deeper meaning form the playing of diketo, as follows:

> The mathematical knowledge/content extracted from this phrase ", *6 stones, we got 12 throws…*", it shows that stones used to play diketo have relationship with the throws of the ghoen. The relationship can be obtained by patterning (that is doubling the number of stones to get the total number of throws made), which ultimately result into the general relationship between stones and the number of throwing the ghoen. Hence, the research participants (learners) were able to demonstrate the general formulae as $f(x) = 2x$. On the other hand, those playing with "pups", they established the general relationship between stones used in the play of diketo and number of throwing the ghoen as $f(x) = 2x - 1$. The established general formulae imply that there are two variables involves, that is the number of stones in this case demonstrated the independent variables and throwing of the ghoen shows the dependent variables.

FINDINGS

The research found that the teachers integrated the mathematical content with the prior knowledge of the learners, which is the playing of cultural games. Teachers used those games played by learners and parents to extract mathematical content embedded in them. It was an interactive session in which all learners worked together to play the cultural games, whilst other participants used the observation sheet to visualise the mathematical skills and content demonstrated. They discovered many mathematical concepts infused in the playing of diketo. Some of the mathematical content embedded are sorting and classifying data in a concise way, that can be easily interpreted. Table 2 above showed how the relationship can be obtained between column 1 and 2, and relationship between column 3 and 4, through patterning, so as to get general formulae. Thus , the general relationship between throwing of the ghoen and scooping of the stones out of the hole (in table2, column 1 and 2 above) can be described as $f(x) = -\frac{x}{2} + \frac{13}{2}$.

Similarly, in table 2 above (column 3 and 4) denotes the relationship between the throws of the ghoen and stones placed back into the hole expressed as $f(x) = -\frac{x}{2} + 6$. This is good play, which allows learners to engage in investigation process and self-discovery. As the DoE (2003) argues, mathematics is a human activity practiced by all cultures. The paper found many mathematical concepts and much content.

CONCLUSION

It was found that the cultural capital possessed by learners assisted in concretizing the mathematical content, which initially looked abstract. Teachers need to allow learners to explore general formulas or mathematical procedures in various ways. Learners who played diketo using "pups" and "without pups "came up with logical conclusions, as shown in table 1 and 2 above. In determining the relationships between variables, patterning can be used to get general relationship. It was demonstrated that formulae can be crafted from observing a particular sequences of occurrences during the play of diketo. Any approach used to play diketo helped leaners to realise variety of reasonable general conclusions drawn. It was interesting to observe that learners demonstrated why the independent and dependent variables related and defined.

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WORKSHOPS

STRATEGIES FOR PROVING AND SOLVING EUCLIDEAN GEOMETRY RIDERS

Mabotja Tlou Robert

Capricorn District: Limpopo Province

TARGET AUDIENCE: Grade 11 Educators

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 40

MOTIVATION

With regard to Curriculum and Assessment Policy Statement (CAPS), Euclidean geometry in grade 10, 11 and 12 is compulsory and examinable. Many mathematics teachers find it very difficult to teach Euclidean because many were not taught nor had studied it beyond school level. On the other hand, others had not study Euclidean geometry at universities and now are faced with the challenge of teaching concepts which they are not comfortable with. Euclidean geometry problems have created a lot of discomfort in mathematics for both teachers and learners. Therefore, this workshop is aimed at assisting mathematics teachers with skills of proving and solving Euclidean geometry riders. Emphasis will be placed on diagram analysis, identification and naming of theorems.

CONTENT

Curriculum and Assessment Policy Statement (CAPS) with regard to Euclidean geometry in grade 11 expects the following:

- Investigation and proving of theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles.
- Solve circle geometry problems, providing reasons for statements when required.
- Prove riders.

DESCRIPTION OF CONTENT

In this workshop, we will be focusing on proving and solving of Euclidean geometry problems through the use of the following strategies:

ACTIVITY	CONTENT	TIME FRAME
1	Motivation and background	10 minutes
2	Chord and Radii of circles	10 minutes
3	Circle angles, inscribed angles and angles in the same segment	20 minutes
4	Diagram analysis	20 minutes
5	Cyclic quadrilaterals	30 minutes
6	Tangents	20 minutes
6	Conclusion and findings	10 minutes

CONCLUSION AND FINDINGS

Diagnostic report for 2017 final examination paper has outlined the following suggestions for improvement:

The proofs of the theorems should be introduced only after a number of numerical and literal riders.

- Teachers must cover the basic concepts.
- They need to assist learners in naming angles correctly.
- Learners should be taught that all statements must be accompanied by reasons.
- They should also be taught to use acceptable reasons.
- Diagram analysis must be emphasised.
- Teachers should be encouraged to scrutinise the given information and the diagram for clues about which theorem could be used in answering the questions.
- Teachers need to be told that there is no short cut to mastering the skills required in answering questions on Euclidean geometry.
- The teaching of theorems should be done with the relevant understanding.

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VARIABLE MEMORIES AND SOLVING EQUATIONS USING A CASIO SCIENTIFIC CALCULATOR

Astrid Scheiber

CASIO

TARGET AUDIENCE: Further Education & Training

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 60

MOTIVATION

Adequate knowledge of calculator skills makes the teaching of Financial Maths & Functions easier and enables the educator to assist their learners more efficiently. This workshop will guide you through the calculator functions, using the

CASIO FX-82ZA PLUS Scientific calculator.

CONTENT

This workshop will cover: In-putting values into the CASIO calculator MEMORY, using the saved values & recalling what has been saved. Using TABLE MODE – solving Simultaneous, Quadratic & Cubic equations.

Introduction	5 minutes
In-putting values into the calculator memory	5 minutes
Using the saved values	5 minutes
Recalling what has been saved	5 minutes
Financial Maths – 2 worked examples - Future Annuities	20 minutes
Using TABLE MODE to find points of intersection of a straight line & parabola and turning point of the parabola	25 minutes
Solve Simultaneous equations & worked example	15 minutes
Find the roots of a Quadratic equation & worked example	15 minutes
Find the roots of a Cubic equation & worked example	15 minutes
Discussion	10 minutes

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TRANSFORMATIONS

Sekgana Motimele

Waterberg District Office

TARGET AUDIENCE: Intermediate Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 60

MOTIVATION

I realised that as educators it is very important for us to have conceptual understanding of the concept, which will in turn help us to improve the way we teach the topic to our learners. The topic is very important as it forms the basis for FET where learners perform poorly as far as transformation of graphs are concerned.

DESCRIPTION OF CONTENT

- Tessellation will be done using manipulatives
- Rotation (Turn) will be done using patty paper
- Translation (Slide) will also be done using patty paper
- Reflection (Flip) will be done using a mirror where the concept of mirror image will be developed. Patty paper and paper folding will also be used here.
- Symmetry and reflection will be shown as having a special relationship

WHAT WILL BE DONE IN THE WORKSHOP

Introduction (10 minutes)
Activity 1 (25 minutes)
Activity 2 (25 minutes)
Activity 3 (30 minutes)
Activity 4 (20 minutes)

LEVEL, NATURE AND CONTENT OF THE WORKSHOP

The focus of the workshop is the intermediate phase educators, wherein the content area is transformation geometry. The objective is to capacitate our intermediate colleagues who still struggle with presentation of the section to learners. The other important point worth mentioning is that, since most of our learners remembers more when they do activities than when told, we want to share that as colleagues. In the workshop we will be using things such as the mirror to explain the concept of reflection, while at the same time we are bringing home the reason why we have words such as mirror image etc. The other thing that will be done is where simple objects like patty paper are used to explain concepts that would otherwise be difficult for learners. The other important point is where we show that as educators we don‟t only need complicated things to explain concepts to learners, but as educators we can use everyday materials as our teaching aids.

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CAPS Policy Document (Mathematics Intermediate Phase)

www.youtube.com

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FUN ACTIVITIES TO TEACH 3D OBJECTS

Liezel du Toit and Amanda van der Merwe

Institute for Professional Development

TARGET AUDIENCE: Intermediate Phase teachers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 40

MOTIVATION

Learners often struggle with this content area of Space and Shape (Geometry). This is a practical workshop to assist the teachers to teach the different 3D objects in a fun way. This will help the learners to remember what was taught. Teaching and learning will be enhanced and results should improve.

DESCRIPTION OF WORKSHOP CONTENT

1. Start the workshop with a Mental Mathematical activity to practice and consolidate 3D objects. (10 minutes)
2. Overview of how to recognise and identify 3D objects. (20 minutes)
3. Build 3D objects (prisms and pyramids) with cool drink straws. (40 minutes)
4. An activity to help the learners to visualize the 3D objects. (30 minutes)
5. Discussion and reflection on the activities. The aim will be to discuss if and how these activities will „work‟ in the classroom. (20 minutes)

Total: 2 hours

LEVEL, NATURE AND CONTENT OF THE WORKSHOP

The study of Space and Shape improves understanding and appreciation of the pattern, precision, achievement and beauty in natural and cultural forms. It focuses on the properties, relationships, orientations, positions and transformations of two-dimensional shapes and three-dimensional objects. (Curriculum and Assessment Policy Statement for Mathematics page 10.)

Play is an important part of early childhood learning. Even very young children observe geometrical patterns. Opportunities to explore shapes are important to encourage this development. This development already starts in pre-school years and continues. An important part of developing geometric understanding is to investigate the properties that are common to all shapes in a particular category. (Siemen et.al, 2013). Visualisation is an important aspect of geometry and mathematics. It is more than just „seeing images in the mind". Rather, „spatial visualisation is the ability to generate and manipulate images". (Clemens & Samare, 2007, p. 499)

In this workshop we aim to clearly define what is meant by 3D objects. We will discuss an overview of the classification of 3D objects to help learners to recognize and identify 3D objects. We will apply our knowledge when we visualise and name different 3D objects. During the workshop we will aim to make the whole concept concrete when building prisms and pyramids. We strive to work from concrete to semi-concrete and then abstract to support the learners to visualise the objects visually. We will describe, compare and sort 3D objects based on features that are similar or different. We will match nets to 3D objects.

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TEACHING A NEW LANGUAGE - ALGEBRA

Liezel du Toit and Amanda van der Merwe

Institute for Professional Development

TARGET AUDIENCE: Grade 7, 8 and 9 teachers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 40

MOTIVATION

Algebra is part of the content area of Numbers, Operations and Relationships. The ability to construct algebraic text to describe relationships is an area of difficulty for many learners in the Senior Phase. Algebra has been described as generalized arithmetic. It is used to capture important number properties that are true regardless of the number chosen. But algebra can also be used to represent variables in a general relationship such as cost of hire a car etc.

Learners are struggling with all these concepts. The aim of this workshop is to introduce algebraic language to learners as well as do calculations with algebraic expressions such as revise conventions, addition, subtraction, multiplication, factorisation and division.

DESCRIPTION OF WORKSHOP CONTENT

1. Ice-breaker activity: In groups discuss where we use numbers in our everyday life. Then they have to translate this into algebra terms. (10 minutes)
2. Fun ways to introduce expressions, terms and factors to the learners. Learners need to "think algebra". (10 minutes)
3. Calculations with algebraic expressions (35 minutes)
4. Factorisation, a very important skill for grade 9 building into FET. (20 minutes)
5. Division (15 minutes)
6. Reflection, discussion and feedback: How will these activities work in the classroom. (30 minutes)

Total: 2 hours

THE LEVEL, NATURE AND CONTENT OF THE WORKSHOP

Algebraic thinking is foundational to all areas of mathematics because it provides the language and structure for representing and analysing quantitative relationships. The relationships are needed for modelling situations, solving problems, and proving generalisations. Algebraic thinking has the capacity to assist in unifying concepts by allowing us to express similarities and differences in a way that helps us see the commonalities across the school curriculum, grade levels and mathematical settings. (Siemon et. Al. 2013. Page 537). Is Algebra important? According to the Australian Academy of Science (AAS, 2003) generalising and mastering challenging mathematical problems, applying mathematics to real-world problems and the ability to use technology as a tool for conjecturing and generalising, underpin economic success and innovation in a global knowledge economy. The shortage of scientists and mathematicians is a concern.

Letters in algebra are used as symbols to represent unknowns and variables, and this is what learners find problematic. With this workshop we aim to provide practical examples and activities on how to make the teaching easier and fun. We focus on the introduction of algebraic language to learners. We will revise the conventions in algebra and then focus on different calculations with algebraic expressions, namely addition, subtraction, multiplication, factorisation and division. These activities will provide the teachers with activities for the classroom to improve learning and teaching to improve results.

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USING A SPREADSHEET TO ENHANCE MATHEMATICAL THINKING

Patisizwe Mahlabela, Nomathamsanqa Mahlobo and Themba Ndaba

Centre for the Advancement of Science and Mathematics Education (CASME)

TARGET AUDIENCE: Senior Phase

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Some of the specific aims of teaching and learning mathematics outlined in the Curriculum and Assessment Policy Statement (CAPS) are to develop in learners:

- an appreciation for the beauty and elegance of Mathematics
- a spirit of curiosity and a love for Mathematics (p. 8).

These are also the aims of this workshop. CAPS (p. 9) proceeds to outline, amongst others, the ability to investigate, analyse, represent and interpret information as the skills that should be developed in learners. The workshop also seeks to do exactly that. Mason, Burton and Stacey (2010) suggest that mathematics should be taught such that it stimulates in learners thinking mathematically. Thinking mathematically entails doing investigations and coming up with conjectures. It entails making generalisations and justifying them. The workshop, using some activities from their book, aims to show how spreadsheets could be used to enhance thinking mathematically.

Teachers often make excuses for not using investigative methods to teach mathematics. They are of the opinion that investigations are time consuming. Use of spreadsheet nullifies this excuse. Computers and calculators are fast and accurate. Lastly, there is a notion that learners live in a world of technology. When they come to our classrooms, they often found themselves in a different world because most teachers do not use technology to teach. The workshop wishes to promote use of technology in the teaching of mathematics.

DESCRIPTION OF WORKSHOP CONTENT

- Description of the spreadsheet
- Entering formulae onto a spreadsheet
- Using the spreadsheet to generate patterns
- Determining general terms of a patterns generated
- Justifying generalisations made.

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TEACHING GEOMETRY THROUGH APPRECIATION OF THE BEAUTY AND MATHEMATICS OF SYMMETRY

Michelle Du Toit and John Lawton

Education Resources Africa (ERA), Objective Learning Materials (OLM)
Australia

TARGET AUDIENCE: Teachers of mathematics for years 4 to 9 and beyond

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 100

MOTIVATION

Geometry is a difficult subject for many mathematics teachers, and students. Van Hiele theory(Ekanayake, Brown, & Chinnappan, 2003) suggests that geometry students need to learn by acquiring entirely new ways of reasoning, not just by learning to implement the same solution strategy more effectively, and that this presents unique challenges to geometry educators. Studies such as those by Senk (1989) found that in all years of schooling learners are scattered across a range of reasoning types, so that it becomes impossible for teachers to use any single learning strategy with an entire class.

Active learning, using the MATHOMAT template as a tool, offers a means for engaging with the creative challenge in teaching geometry. Mathomat can be used intuitively to explore concepts in geometry as a sequence of actions (both mental and physical). These actions carry within themselves the seeds for powerful ideas which can be drawn out and studied in scientific terms as learners attain higher levels of abstraction, under teacher guidance. See research by Tall (2000) for an explanation of how active learning tools help in understanding geometry through intuitive learning.

Active learning can only occur in context, and should be based on teaching that is both rigorous conceptually and meaningful for students. In this workshop we address geometry through the study of the symmetry in everyday objects around us. We learn, initially through intuitive actions with Mathomat but then by inquiring about the properties of Mathomat as a tool and the objects under study to work conceptually with the underlying mathematics of their symmetry.

Numeracy is a crucially important aspect of the curriculum. We are in a computerised, globalised, world in which, to be successful, communities must have all of their members able to work mathematically. It is unacceptable for schools to deal superficially with the numeracy challenge. In our workshop we address how to achieve a deep and flexible understanding of an aspect of an often difficult topic (geometry) through appreciation of the mathematics of symmetry in the world around us. This leaves learners with a richer understanding of their world, able to understand its mathematical structures, to engage in skilled design work, and to appreciate and build on its culture more effectively.

DESCRIPTION OF WORKSHOP CONTENT

Learners in this session engage in creative design work using Mathomat templates as a tool. It is important for learners to work with their hands throughout this session, and to achieve a sense of ownership of the content early on. This is achieved by asking learners to consider examples of interesting, and inspiring, 2-D patterns in the Mathomat materials available in the session, and to then experience a sense of personal satisfaction from developing their own Mathomat drawings.

Learners are then introduced to the four isometries of the plane (reflection, rotation, translation and glide reflection) as action sequences that can fully explain the transformation of a motif so that it retains its original size and structure. These isometries are the underlying mathematics in the creative drawings done earlier (assuming that students follow the example of the tessellation drawings shown early in their creative efforts). Learners are encouraged to study one of these isometries, of their own choosing, and to then use their Mathomat to create a shape and transform it in this way; then writing about their understanding of that transformation.

After John Lawton explains how reflection is a building block for the other isometries, learners are introduced to the idea that two of the isometries, under certain conditions, can be used to transform a 2-D shape into itself. These are *rotation* around a *centre* in the middle of the object and *reflection* across a mirror line. Learners are then asked to find lines of symmetry and rotational symmetries in real world objects using their Mathomat templates, and Mathomat published hand-outs. The room is then split into groups of two types (A&B). A group are asked to classify the shapes in their Mathomat templates according to the number of lines of symmetry in each, while B group are asked to find how many rotational symmetries there are in each of the shapes in their Mathomat templates. Both groups are asked to report on their findings by presenting a poster of results.

John Lawton introduces the idea that every finite 2-D object belongs to one of two possible symmetry groups: the Dihedral (or two-faced) group denoted Dn and the Cyclic (because it goes round in a circle) group, denoted Cn. Learners are asked to classify the shapes studied in earlier handouts, and the (non-circular) shapes in their Mathomat templates according to the total of their potential symmetry operations.

Wrap up and discussion in the session follows, led by Michelle du Toit. The role of symmetry in teaching geometry, its importance to understanding the structure of both natural and built environments will be addressed; discussion will also cover the role of tools in learning and the ability of students to use tools more skilfully if their underlying symmetry characteristics are understood.

The overall structure for this workshop was developed by John Lawton, and is based on his experience with the topic *Patterns in the plane* from the unit *Exploring space and number* in the Master of Education course at Deakin University as studied by John Lawton. Core concepts in this workshop are derived directly from that program, including some of the presentations, related to the isometries of the plane, symmetry operations and scientific classification of objects according to group theory. The way that Mathomat is related to these concepts as a tool was developed by John Lawton as the senior designer of the Mathomat template, by John Lawton and Juliet Snape as authors of the Mathomat instruction book, and by Susie Groves and Peter Grover as authors of the lesson plan series *Maths with Mathomat* (published by Objective Learning Materials).

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ROLE OF VISUALISATION IN THE TEACHING AND LEARNING OF MATHEMATICS: A CASE OF SOLVING NON-ROUTINE PROBLEMS

Clemence Chikiwa

Education Department, Rhodes University, South Africa

TARGET AUDIENCE: Intermediate, Senior and FET Phases

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 40 (10 groups of four)

MOTIVATION

Research has shown that teaching and learning of mathematics can be greatly enhanced through the incorporation of visualisation processes in education (Arcavi, 2003; Presmeg, 2006; Rivera, Steinbring & Arcavi, 2014). Visualisation according to Arcavi (2003) occurs at various levels and stages. He defines visualisation as:

> … the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings (p. 217).

Visualisation occurs at creation level of pictures and images, when interpreting them, during use of these, and when reflecting upon various representations. This workshop is aimed at practically exploring the role of visualisation and encouraging teachers to use various visualisation techniques when solving mathematical problems. Presmeg (2006) views visualisation as an aid to understanding, thus this workshop will be presented with the assumption that to visualise a problem means to understand that problem in terms of visual images or representations. Boaler (2017) advises that mathematics is a subject that allows for precise thinking, but when that precise thinking is combined with creativity, visualization, and flexibility, the mathematics comes alive. Thus, there will be a deliberate move to encourage participants to create and form visual models that will best assist in solving given non-routine word problems.

NON-ROUTINE PROBLEMS

Non-routine problems are those for which no straight forward prior or known algorithm exists, or is thought to exist at a particular level of learning, that can provide the solution to the problem. Non-routine problems are identified by the features of openness, critical thinking and novelty (Ministry of Education, Jamaica, 2011). Thus they have a tremendous potential to impact students" ability to adapt to real-world situations. Solving non-routine problems involves the creation of strategies in order to get to an intended end. In day-to-day life, real-world problems rarely come as nicely packaged and highly abstracted as those that learners frequently encounter in routine classroom problems; rather, they are characterized by ambiguity, missing or unknown information and are usually solved after much analysis and critical thinking (Santos-Trigo & Camacho-Machin, 2009). A school curriculum of predominantly or only routine problems, therefore, ill-prepare learners for excelling in real life situations where strategizing and critical thinking will be needed. This thus foregrounds the need for carefully and seriously considering non-routine problems as an important component of mathematics teaching and learning.

BENEFITS TO PARTICIPANTS

A picture, diagram or image speaks a thousand words. Visualisation when appropriately used, acts as a knowledge compression. Visual chunking, a practice where bits of information that are related in some way are combined in order to reduce the overall amount of information for easier and faster processing, is one way teachers use in classrooms. It thus helps problem solvers to free working memory space in the brain thereby enhancing efficient and presumably faster ways of solving problems. Interest is to promote solving of mathematical tasks visually during teaching and learning. The workshop is intended to help equip participants with simple yet effective ways of solving non-routine mathematical problems. It will also be used to help as a catalyst for participants to later seek more appropriate ways to efficiently and effectively solve mathematical problems visually. There is accumulating evidence linking visualisation processes to deep understanding of key concepts and fundamental principles in various mathematical domains (Abdullah et al., 2012). Visualisation permits deep learning and understanding due to its ability to embed the foundations of mathematics in its processes while making logical links to other areas of the larger mathematical curriculum. The use of visualization processes in the study of mathematics provides learners with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of all mathematical concepts at school level.

The problems chosen for this workshop are tailored towards encouraging participants to explore a variety of visual solutions strategically and to think critically. As noted by Van de Walle, Karp and Bay-Williams (2010, 32), "most, if not all important mathematics concepts and procedures can best be taught through problem-solving”. Teaching through non-routine problem solving encourages teachers to ask questions that are open and that facilitate critical thinking.

DESCRIPTION OF WORKSHOP CONTENT

The workshop will comprise of tasks that cater for different phases of school learning. Selected non-routine open-ended tasks will be solved through the incorporation of visual techniques and processes. Teachers of different school phases will be encouraged to mix. Problems given will be of non-routine nature that can be solved by various methods depending on the level at which one is teaching. Hence teachers will be required to compare their visual techniques.

CONCLUSION

Visualisation permits deep learning and conceptual understanding due to its ability to embed the foundations of mathematics in its processes while making logical links to various areas of the larger mathematical curriculum. This workshop serves to encourage and make teachers aware of the crucial role visualisation processes play in the teaching and learning of mathematics. This will be achieved by use of carefully selected non-routine tasks that will be solved during the workshop. This workshop will help equip participants with simple yet effective ways of solving non-routine mathematical tasks. It will also be used as a catalyst for participants to later seek more appropriate ways to efficiently and effectively solve mathematical problems visually. There is accumulating evidence linking visualisation processes to deep and long lasting conceptual understanding of key concepts and fundamental principles in various mathematical domains.

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APPLICATION OF CALCULUS WITH REFERENCE TO SCHOOL MATHEMATICS

Themba Ndaba, Nomathamsanqa Mahlobo and Mvunyelwa Msomi

Centre for the Advancement of Science and Mathematics Education (CASME)

TARGET AUDIENCE: Further Education and Training (FET) Teachers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 30

ABSTRACT

This workshop serves to capacitate participants who are involved with teaching of calculus as a topic. Participants will first be introduced to basic theories of calculus, like the concept of a limit. The focus of the workshop will be to emphasise the role played by maximizing and minimising in real life. Participants" attention will be drawn into the importance of knowledge of areas, volumes and calculations of these with different shapes and dimensions. Participants will be provided with five (5) exercises on application of calculus from which will be followed by debates, discussions and consolidation.

RATIONALE

Application of calculus has proven to be the section in which most learners underperform. This is clearly shown in the diagnostic report 2017. Only 9% of learners who wrote Mathematics in 2017 got question 9, which was about the application of optimization in calculus, correct. Therefore, the workshop will serve to address the situation.

CONCLUSION

The participants will have to state their experiences coupled with the solutions of the tasks and these will be discussed and debated.

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THE PEDAGOGY OF MATHEMATICS IN SOUTH AFRICA: IS THERE A UNIFYING LOGIC?

Lindiwe Tshuma, Nicky Roberts, Mogege Mosimege

AIMSSEC, University of Johannesburg, University of the Free State

TARGET AUDIENCE: Primary mathematics teachers, Lecturers – Primary Mathematics Education, Mathematics education researchers, In-service teacher training providers

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 60

ABSTRACT

Mathematics is a loaded term: society seems to divide between those few who experience sheer joy from the subject, and the majority for whom it is a phobia. Nations obsess about their ability to impart and absorb mathematical knowledge; global comparative studies on the matter are taken as grounds for national pride or shame. Post-apartheid South Africa has had its own share of self-flagellation, with much research into the reasons behind the country‟s poor comparative performance.

A recent book on *The Pedagogy of Mathematics in South Africa* offers an overview of that research, re-asserting some of the findings of previous studies. These include evidence that the state of mathematics teaching and performance in South Africa today reflects the impact of its colonial and apartheid past, and a racist system that presupposed that keeping mathematical knowledge from the oppressed would prove their supposed inferiority. However, this book goes beyond historical issues to pose crucial questions: why at all do we teach mathematics? What is the subject‟s actual utility to life? And there is a unifying logic informing our South African way of teaching mathematics?

The release of the book was considered a starting point for further engagement with South African stakeholder in mathematics education on the elements that bind and unite us in relation to how we approach the teaching of mathematics.

Join some of the contributing authors to the book to explore:

- Building consensus on the key characteristics of our hoped for national pedagogy
- Word charts and examples of the multiple meanings of everyday terms and their use in the mathematics register

- Generating and exploring ethno-mathematics examples to integrate into our South African classrooms
- Developing your own personal history of mathematics in South Africa

MOTIVATION

Mathematics is a loaded term: society seems to divide between those few who experience sheer joy from the subject, and the majority for whom it is a phobia. Nations obsess about their ability to impart and absorb mathematical knowledge; global comparative studies on the matter are taken as grounds for national pride or shame. Post-apartheid South Africa has had its own share of self-flagellation, with much research into the reasons behind the country‟s poor comparative performance.

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The release of the book was considered a starting point for further engagement with South African stakeholder in mathematics education on the elements that bind and unite us in relation to how we approach the teaching of mathematics.

DESCRIPTION OF THE WORKSHOP CONTENT

In seeking to answer the above questions, the authors of the *Pedagogy of Mathematics in South Africa* explore some of the best practices in mathematics education, both locally and internationally. They argue for possible methods of nurturing mathematical thinking amongst young people in South Africa.

Key issues to emerge are the importance of teaching mathematics in a way that links to learners‟ concrete social environment, and the necessity for joint efforts on the part of government, unions and private partners. In addition, the study argues for the importance of teachers‟ developing a deeper understanding of mathematics, and of creating learners with productive mathematical identities, capable of making sense of mathematics in South Africa‟s diverse languages.

The workshop will be structured around engagement and discussions with particular exemplar topics which feature in the book:

Nicky Roberts: *Introduction and overview to the pedagogy of Mathematics in South Africa*

Key activity: What do we each imagine as key characteristics of our national pedagogy?

Dr Lindiwe Tshuma: *Language as a Resource in Intermediate Phase*

Key activity: Word charts and examples of the multiple meanings of everyday terms and their use in the mathematics register

Mogege Mosimege: *An Ethnomathematical Approach to Mathematics Teaching and Learning in South Africa*

Key activity: Generating and exploring ethnomathematics examples to integrate into our South African classrooms

Nicky Roberts :*A Historical and Socio-political Perspective on Mathematics in South Africa*

Key activity: Developing your own personal history of mathematics in South Africa

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NGOKUBA SIYAZI LENA____, LENA ILULA ______

Ingrid Mostert and Zikhona Gqibani

Axium Education

TARGET AUDIENCE: Foundation phase teachers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: no maximum

ABSTRACT

Because we know how to do this: _______, this is easy: ________. When we teach mathematics there are many opportunities for learners to use the things that they already know (e.g. 12 + 12 = 24) to work out things they don‟t know (e.g. 12 + 13 = ?). This workshop will teach you four strategies that your foundation and intermediate phase learners can use to do calculations more quickly and flexibly. You will work in a small group to practice the strategy and to make your own examples of calculations where you can use this strategy.

MOTIVATION

Although CAPS advocates that learners are exposed to a number of different strategies for doing calculations with whole numbers in the foundation phase, often learners are taught one strategy as „the‟ way of doing addition, subtraction, multiplication or division. Such an algorithmic approach to teaching mathematics provides learners with little opportunity to practice using what they already know to find out something new. This workshop introduces participants to a number of cases where facts that learners typically know can be used to determine other facts.

DESCRIPTION

The focus of the workshop will be to allow participants ample opportunity to engage with mathematical problems where known facts can be used to simplify calculations. Once participants are comfortable using a particular strategy, they will practice generating examples of calculations which can be solved using that particular strategy.

The four strategies that will be explored are:

- Using adding 10 to add 9 (because 9 is one less than 10) and using adding 100 to add 99.
- Using addition of units (e.g. 3 + 4 = 7) to add tens, hundreds, etc. (e.g. 30 + 40 = 70 and 300 + 400 = 700). This will be done with a Gattengo chart.

- Using doubles to calculate near doubles.
(e.g. 8 + 9 = 8 + 8 + 1 = 16 + 1 = 17)
- Using doubling to calculate 4 and 8 times tables as well as other multiplication strategies.

Participants will work through each of the cases in small groups (3-5 participants). After a number of problems have been solved for a particular case, groups will have some time to generate their own examples and to share these with the bigger group.

15 min: Introduction and discussion about the importance of helping learners to become flexible in how to do calculations

15 min: Adding 10 and adding 100 is easy

15 min: Introduction to Gattegno chart

15 min: Using Gattegno chart to add tens, hundreds, decimals etc.

15 min: Using doubles to calculate near doubles

15 min: The difference between „3 more than" and „3 times more than"

15 min: Using known multiplication facts to figure out other multiplication facts

15 min: Final discussion

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TESSELLATIONS: MANIPULATING & EXPLORING SHAPES CREATIVELY AND MATHEMATICALLY

Zonia Jooste , Stanley Adendorff

Cape Peninsula University of Technology (CPUT)

Rajendran Govender

University of the Western Cape

TARGET AUDIENCE: Intermediate and Senior Phase Teachers/Lecturers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 35

MOTIVATION

We, as lecturers have come to the realization that tertiary level students" knowledge of tessellations is limited, which could be proof that the topic is possibly not taught effectively in some schools. We therefore wish to highlight and emphasize the significance and merit of teaching and learning tessellations, a topic that enhances an appreciation for the beauty and marvel of mathematics.

DESCRIPTION OF WORKSHOP CONTENT

- Participants will complete a short baseline activity to ascertain their basic knowledge levels [10 minutes]
- Participants will be given shapes (models) to do exploratory activities and complete a table [15 minutes)
- Discussion on the findings in (2) [10 minutes].
- Social knowledge discussion: What is tessellation? [5 minutes]
- Measurement activity: exploring through measurement why some regular 2-D shapes tessellate and why others do not. Discussion of vertex arrangements in tessellations [20 minutes].
- Discussion of different types of tessellations – using two or more shapes to form tessellations [10 minutes]
- Discussion: ideas for classroom implementation [5 minutes]
- Instructions for creating authentic tessellations through transformations and Escher methods [10 minutes]
- Activity: participants create their own unique tessellations [35 minutes]

- Reflection

Acknowledgement

higher education & training
Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

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THE THING ABOUT COMPREHENSION AND MATHS

Connie Skelton

Iconic Maths

TARGET AUDIENCE: Foundation, Intermediate and/or Senior Phase

DURATION: 2-hour hands-on workshop

MAXIMUM NUMBER OF PARTICIPANTS: 50

ABSTRACT

Mathematics is a wonderful, complicated and sometimes frustrating and frightening subject to many learners. Part of its mystery comes from learners“ beliefs that mathematics involves only numbers, abstract symbols, and their interrelationships. Learners forget that mathematics involves common sense and language abilities too.

Learners develop reading and mathematics skills at different rates. Some learners develop algorithm skills (i.e., the ability to compute) quite well until they are faced with word problems. For those learners, it is ordinary language in mathematics, and the format of instruction, that presents difficulty. Therefore, to solve mathematical problems, which are a common part of the school curriculum, such learners need to learn to read.

To improve their mathematics, we must improve their reading.

MOTIVATION

Although many people believe that mathematics only involves numbers, abstract symbols and their interrelationships, it also requires language and common sense. Apart from the problem-solving aspect where reading is very important, the methods of instruction also require reading comprehension. To improve their mathematics, we must improve reading.

DESCRIPTION OF WORKSHOP CONTENT

Introduction	10 minutes
What is reading comprehension?	3 minutes
What is mathematics?	2 minutes
Activity 1	15 minutes
Activity 2 Algebraic thinking	10 minutes
Other important aspects of comprehension	5 minutes
Strategies for teaching reading comprehension skills	5 minutes
Activity 3	15 minutes

Learning to read word problems	10 minutes
Activity 4	15 minutes
Activity 5	15 minutes
Conclusion	5 minutes

WHAT IS READING COMPREHENSION AND MATHEMATICS?

Reading comprehension separates passive unskilled readers from the active readers. Skilled readers do not just read; they interact with the text.

Mathematics:

- is about theory.
- is the study of relationships between numbers, shapes and quantities?
- uses language, symbols, signs and proof.
- also uses mental mathematics, algebra, geometry, trigonometry, calculus, etc.

CONCLUSION

A very important milestone in learners‟ education is when they go from learning to read to reading to learn. Good reading comprehension opens the door to acquiring new knowledge. Better reading comprehension has a positive impact on learners‟ academic performance in all subjects where reading is the main source of information – including mathematics. Many studies show a relationship between certain types of linguistic abilities and mathematical abilities (see, e.g., Zhang, Koponen, Räsänen, Aunola, Lerkkanen and Nurmi, 2014). Ideally, problem solving should also be based on contexts that learners are familiar with and can relate to. To improve their mathematics, we must improve their reading.

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USING NUMBER LINE TO DECONSTRUCT MATHEMATICS PROBLEMS IN FET

Julius Olubodun

ORT South Africa

TARGET AUDIENCE: FET teachers

DURATION: 2 hours

NUMBER OF PARTICIPANTS: up to 50

ABSTRACT

In ORT South Africa, we strive to empower and support teachers to get maximum value for their time in and out of the classroom – learners doing mathematics and making sense of it. During our Classroom Based Supports (CBS) we often witness FET teachers trying to engage their learners productively, with little or no success. We had to look for a way to get the learners to use what they already have or know (that is their lower grade expertise) in order to make sense of such mathematics concepts as Sequence and Series (arithmetic, geometric, exponential); Financial Mathematics; Analytical Geometry; and Algebraic expressions involving Linear and Quadratic problems (both equations and inequalities). (National Research Council, 2001, p. 87; Van de Walle, Karp, & Bay-Williams, 2010, p. 20 and 24)

Number line came to our rescue. This workshop will empower FET teachers to help their learners work with the number line and to use it to deconstruct mathematical problems and work through to solving the problems. We hope our engagements during the workshop will lead to more discoveries, by the participants, of other areas where their leaners can apply these strategies involving number line. (National Research Council, 2001, p. 87)

MOTIVATION

Teachers of mathematics in FET are often faced with many learners in their classrooms whose proficiency in mathematics are those of the lower grades. Many of these learners are able to relate with mathematical abstraction of lower grades while they are presented with mathematics of higher levels of abstraction in FET. On many occasions, the mathematics teacher asks questions, but when these learners fail to find any connections between what they know and what they have to do, the lesson is dominated by silence with only a few learners making intermittent contributions. The teacher ends up answering the question himself. This session will equip the participants with strategies to create opportunities for all learners to deconstruct new mathematics challenges using the number line.

Mathematics (or numeracy) learners in South Africa began to engage with number line since foundation phase. (Department of Education, 2011, p. 20). The ruler as a tool that learners are conversant with on a daily basis as well as the stair case (in some cases the elevator – though this may be remote to some learners) can help bridge any gaps in their knowledge of number line.

CONTENT OF THE WORKSHOP

Activity 1: Integer representations; addition and subtraction with number line. (30 minutes)

Activity 2: Sequence and Series (arithmetic, geometric, exponential) – Solve selected NSC questions with number line. (30 minutes)

Activity 3: Analytical Geometry – Solve selected NSC questions with number line. (30 minutes)

Discussions (30 minutes)

CONCLUSION

This workshop will help teachers to create meaningful learning experiences for each and every learner in their classrooms irrespective of the level of proficiency in mathematics that these learners bring to the FET. (Novak, 2002).

With the skills to use and manage this tool, teachers will be able to empower their learners to deconstruct problems and find solution using relationships they can see on the number line.

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FINANCIAL MATHEMATICS IN MATHEMATICAL LITERACY MADE EASY BY USING THE CASIO FX-82ZA+ NATURAL TEXTBOOK DISPLAY CALCULATOR.

Mokonyane "*Mack*" Lediha

Casio

TARGET AUDIENCE: Mathematical Literacy teachers

DURATION: 1 hour

MOTIVATION

As stated by North Carolina (NC) State University, Financial Mathematics is the application of mathematical methods to financial problems. It draws on tools from probability, statistics, stochastic processes, and economic theory (2018). The use of a Casio scientific calculator can enable the teacher or learner when all the functions of the Casio scientific calculator are maximized. It will help equip both teacher and learner in improving their calculator skills, thus making teaching more effective and efficient in the class room. Financial Mathematics is important as it is a flourishing area of modern science. Its numerous applications have become vital to the day to day functioning of the world"s financial institutions according to North-West University (NWU) Natural and Agricultural faculty (2018). The workshop will assist participants in utilizing some of the functions they may not have been aware of, to acquire the desired solution in Financial Maths and Mathematical Literacy as a whole.

PLANNING

5 Minutes	Introduction
15 Minutes	Computational Mode
20 Minutes	Table Mode
20 Minutes	Discussion

MALAWIAN BOW ABACUS: PREPARATION AND USE

Weddington Saka and Nicky Roberts

University of Johannesburg

TARGET AUDIENCE: Foundation phase teachers, Lecturers – Early Mathematics Education, Early mathematics researchers, Early mathematics curriculum developers

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

ABSTRACT

„Concrete materials‟ or „manipulatives‟ have long been advocated for as a key way in which to support young children‟s conceptual development of numbers and their relationships. Several researchers have identified the importance of representations in supporting children‟s problem-solving processes in mathematics, referring to these with various terms: representations models, images, and tools. For the purpose of this workshop, we use the term manipulative to denote physical objects, or apparatus which children can arrange and manipulate. The workshop is on effective use of such manipulatives in developing early number concepts. The workshop will be highly participatory as all workshop participants will be engaged in carrying out several activities. Inquiry-based learning approaches will guide the workshop activities. An indigenous Malawian tool – referred to as a bow abacus – and commonly found in the classrooms and homes of rural Malawian children, is discussed. Its use in classrooms was noted during a study by one of the presenters, but the way in which it was used was found to be limited. The workshop therefore explores how the bow abacus can be effectively used. Participants to this workshop will go through the following content; explore the current and modified bow abacus, make the bow abacus using locally available resources, use the bow abacus to support the acquisition of the following number concepts: counting, ordinality (mental number line), cardinality and decomposability, and part-part-whole relationships.

MOTIVATION

This workshop will equip participants with knowledge of how to make a Malawian bow abacus and how to use it in supporting learners understanding of early number concepts. Practicing foundation phase teachers attending this workshop will be equipped with skills to assist learners develop the early number concept.

DESCRIPTION OF THE WORKSHOP CONTENT

This workshop is a follow up of a paper presented by the same authors titled „Manipulatives for early grade whole number and relationships: the potential of the Malawian bow-abacus". In this paper, the authors present an overview of the theoretical debates about the use of manipulatives in early mathematics learning, outlining some of the distinctions between manipulatives, metal imagery and structured and unstructured external representations. The authors discuss an indigenous Malawian tool – referred to as a bow abacus –commonly found in the classrooms and homes of rural Malawian children. Its use in classrooms was noted but the way in which it was used was found to be limited. As a result, drawing on the theoretical framing of the value of structured representations, suggestions to improve the way in which it can be used are made.

The workshop translates the theory in the paper into practice. Participants will go through a process of assembling a bow abacus using locally available resources. Considering how easy the bow abacus is to make, only ten minutes will be spent on making the bow abacus (see worksheet 1). Considering that appropriate use of the tool is useful, thirty minutes will be spent on tool use (see worksheets 2 - 5) and the remaining twenty minutes will be spent on discussions.

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FAMILY MATH CONTENT IN REAL LIFE CONTEXT

Amaria Reynders and Cobus van Breda

Science-for-the-Future, Faculty of Education, University of the Free State

ABSTRACT

The challenges regarding mathematics teaching and learning in South Africa are common knowledge. Shortages of - or no teaching resources - at school or at home, the lack of parent involvement, and the language of teaching and learning that differs from home languages, are prevalent in many South African schools. Research has indicated that these factors, together with teachers‟ content knowledge and teaching skills, are influential factors in successful mathematics teaching and learning, especially in a diverse classroom set-up (Bernstein, 2004:12; CDE, 2014:15). In order to showcase how these fundamental pedagogical issues can be addressed in the classroom situation, a workshop on Data Handling in the Intermediate Phase will be conducted. The UFS Family Math programme is a hands-on, activity-based initiative, focusing on key concepts in mathematics and engaging learners, educators and parents. Whilst adhering to social constructivist principals, one of the key features of the programme is the use of household, easy available and replaceable activity material in constructing knowledge. The CAPS-aligned programme reflects real-life experiences.

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INCORPORATING GAMES IN MATHEMATICS CLASSROOMS

Rethabile Machaka

Mokitlane Primary School

TARGET AUDIENCE: Foundation and Intermediate Phase

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30 participants (5 groups of six)

ABSTRACT

This workshop is for foundation and intermediate phase teachers. One of the teaching and learning strategies some teachers do not take advantage of, is using games in their mathematics classrooms. In many classrooms, the teacher only allows learners to play games as a time filler, for example, when they are busy marking or preparing other things. This results in a negative association with games during teaching and learning, and learners do not learn about the benefits of using such games. This practical, hands-on fun workshop will provide teachers with the benefits of games in mathematics teaching and learning. Participants will learn how to use these games with mathematical content, in their mental mathematics activities and in the teaching process. These games will help teachers to enhance and sustain learners‟ interest in mathematics. In this workshop, we will explore different mathematics games which use the four basic operations.

MOTIVATION

People of all age love to play games that are fun and motivating. Engaging mathematics games can encourage learners to explore number combination, place value, pattern and other important mathematical concepts. Further, they offer opportunities for learners to deepen their mathematical understanding and reasoning. Bernard Oldfeild (1991) states that many mathematics games are valuable for stimulating and encouraging mathematics discussion.

Discussion can be of immense value to the teacher in illuminating the learner‟s mathematical understanding and thinking… teachers need to know what learners find difficult, in order to decide on the appropriate next stage for those learners. Through these discussions when playing mathematics games learners also get the opportunity to learn from one another.

Playing mathematics games encourages strategic thinking, problem solving and develops fluency. They give both teachers and learners to apply their learning in a different context.

DESCRIPTION OF THE WORKSHOP CONTENT

The workshop will comprise of tasks that cater for the foundation and intermediate phase. Selected mathematics games will be played and their benefits will be discussed. Teachers will be encouraged to work in small groups and discuss their moves with peer.

STRUCTURE OF THE WORKSHOP

TIME	ACTIVITIES
2 minutes	Introduction • Brief introduction of the facilitator. Purpose of the workshop
5 minutes	Ice Breaker • Video clip
10 minutes	Brief explanation of what literature say about games in mathematics classrooms.
15 minutes	Scatter board game Roll and solve game
5 minutes	Discussion
15 minutes	Short and Long Table practice game/ I have… game
5 minutes	Discussion
3 minutes	Wrapping up

CONCLUSION

One of the teaching and learning strategies some teachers do not take advantage of is using games in their mathematics classrooms. Teachers need to understand that games in mathematics classroom should not be played for fun but for the benefits of mathematics. The teachers who will be attending this workshop will be equipped with information that will benefit them. The teachers will be given the opportunity to understand their roles when introducing games in their classrooms.

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USING A SCIENTIFIC CALCULATOR FOR SOLVING EXPONENTIAL EQUATIONS IN GRADE 10

Rencia Lourens

Hoërskool Birchleigh

TARGET AUDIENCE: FET Band

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

Learners in Grade 10 is expected to solve exponential equations of the form $ax = b$ where b cannot be written as a power of base a by inspection.

The aim of the workshop is to use the Scientific Calculator as a tool support the learners‟ inspection in order to get them to understand the concepts without being caught up in doing to the same calculation over and over.

DESCRIPTION OF WORKSHOP CONTENT

15 minutes: Introduction to the Scientific Calculator
10 minutes: Table mode
20 minutes: Solving equations
15 minutes: How can this help when I solve equations?

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TOWARDS AN EFFECTIVE PEDAGOGY IN MATHEMATICS

Rafael Mora Galán, Heyner Alberto Osorio Lara, Yoandy De Armas Ojeda

Cuban Mathematics Specialists at the Department of Basic Education

TARGET AUDIENCE: Secondary Mathematics Teachers: Senior Phase and FET

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30-40

ABSTRACT

The present workshop is part of the studies carried out on the improvement of the process of teaching Mathematics in the South African context. It responds to the diagnosed need to develop a culture of Mathematics teaching based on its conceptualization and contextualization. It offers teachers methodological tools that guide them in the implementation of problem-based teaching, which will come to life in the classrooms, while articulating them with the 21st Century skills, causing a development effect in learners and teachers simultaneously. It contains essentially alternatives for the introduction of different contents and mathematical situations, which can be taught throughout the solution of word problems related to the daily life of learners and teachers.

MOTIVATION

The Department of Basic Education has received 6 Cuban Mathematics specialists who are supporting subject advisors and teachers at different levels. They have already written a number of Training Manuals which will be made available to the mathematics community.

Towards an Effective Pedagogy in Mathematics is one of the Manual that they have developed. In this workshop extracts from the Manuals will be used to conduct the Workshop. Meaningful contribution to the effective approaches and methodologies in the teaching and learning of Mathematics will then be explored through some Worksheets. The workshop blends the Cuban experiences and resources with the South African context.

From the UNESCO Declaration, Education 2030 (May 2015) the declaration asserts that "we will ensure that teachers and educators are empowered, adequately recruited, well trained, professionally qualified, motivated and supported within well-resourced, efficient and effectively governed systems" (page 8). ***Towards an Effective Pedagogy in Mathematics*** will make a contribution to this lofty

UNESCO ideal of empowering and supporting subject advisors and teachers within the DBE system.

It is our goal that all South African Mathematics teachers must teach and learn to think mathematically, while they also think mathematically to teach and learn, based on the problem teaching and learning approach. Therefore, “the effectiveness of Mathematics teaching and learning is a function of teachers‟ knowledge and the use of mathematical content, of teachers‟ attention to and work with students, and of students‟ engagement in and use of mathematical tasks” (Kilpatrick et al, 2001:9). Indeed, effective programmes for teacher training and professional development must help teachers understand the mathematics they teach, effective pedagogies for delivering that mathematics as well as understand how their learners learn that mathematics.

DESCRIPTION OF THE WORKSHOP CONTENT

During the first ten minutes of the workshop there will be a brief presentation of the theoretical foundations of problem-based teaching and its advantages in the development of cognitive skills, individual and collective work in the learners. Then there will be a practical activity consisting of analyze and determine ways to teach different contents through this approach, which will last forty minutes, during this period of time the participants will have an active discussion sharing ideas of different ways to address this approach in the classrooms. And finally the conclusions of the workshop will be made jointly by the participants and the presenters in ten minutes.

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APPROACH TRIGONOMETRY THROUGH REAL-LIFE

Miguel Angel Taboos Cruz, Pedro Pérez Campbell, Yadileydi Hernandez Collot, Ariel Pérez Hechevarria, Phillip Dikgomo

Department of Basic Education

TARGET AUDIENCE: FET Band

DURATION: 1 hour

MAXIMUM NUMBER OF PARTICIPANTS: 30

MOTIVATION

This workshop reflects on one of the most important branches in Mathematics, namely Trigonometric and how the teacher can to teach Trigonometric ratios. Taking into account that Mathematics is a subject that is vital for gaining a better perspective on events that occur in the natural world, a keen attitude and aptitude for Mathematics improves critical thinking and promotes problem-solving abilities. One specific area of mathematical and geometrical reasoning is Trigonometry, which studies the properties of triangles. We also look at the possibility to demonstrate how Physical Sciences can contribute to the development of trigonometric knowledge and vice versa, i.e., that trigonometry can also be used for interpreting and solving problems within the Physical Science context. We will deal with practical applications of Trigonometry to real-life situations.

This workshop will show effective approaches to teaching Trigonometry in FET band because it is well known that students at both the high school and tertiary level are not performing well in this field.

DESCRIPTION OF WORKSHOP CONTENT

Activities	Time	Description	Facilitator
1. Introduction of the workshop and Presentation of the trigonometric approach	15 mins	1. Methodological approach on how to teach trigonometric ratios.	M.A.Toboso
2.1 Solve two-dimensional problems involving right-angled triangles. 2.2 Application of Trigonometric ratio in daily life situations 2.3 Application of Trigonometric ratio in Physical Sciences contents	10 mins		P.P. Campbell Y.H Collet

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PRIMARY TEACHER EDUCATION (PRIMTED)

Nicky Roberts, Zanele Ndlovu, Rajen Govender, Jogymol Alex, Simon Tashie, Sharon McAuliffe, Hamsa Venkat

University of Johannesburg, University of KZN, University of the Western Cape, Walter Sisulu University, University of Free State, Cape Peninsula University of Technology, University of the Witwatersrand

TARGET AUDIENCE: Primary mathematics teachers, Lecturers – Primary Mathematics Education, Mathematics education researchers, In-service teacher training providers

DURATION: 2 hours

MAXIMUM NUMBER OF PARTICIPANTS: 60

MOTIVATION

The Primary Teacher Education (PrimTEd) project is a collaboration between all the universities in South Africa to develop sets of common core standards, materials and assessments of knowledge for teaching primary mathematics. It is funded by the European Union and a lead by the Department of Higher Education. Several workstreams: Number and algebra, mathematical thinking, geometry and measurement, assessment and work integrated learning; have been functional since 2017.

Teacher and mathematics educator engagement with this emerging body of work will enrich the PrimTEd process, and be of value to participants as progress made with regard to common assessments, materials, and theoretical frameworks will be shared.

The results of the 2017 assessment will be shared, which reveals particularly poor performance relating to rational numbers (decimals, percentages, common fractions, ratio)

DESCRIPTION OF THE WORKSHOP CONTENT

The workshop will be structured to first provide an overview of the PrimTEd assessment work stream, and then to break into different group where hand on activities will be initiated by various work stream members.

Participants will rotate through these activities, before coming together for a plenary discussion on PrimTEd and their experiences in each activity. Examples of activities will include tasks relating to multiplying and dividing (multiplicative reasoning) which can draw on the use of powerful representations:

1. Number lines,
2. Double number lines
3. Arrays
4. Grids/ rectangles
5. T-tables/ clue boards:

The group work will involve working on tasks (individually and in groups) and working from the problem situation to develop and use a particular „powerful representations". The workshop is designed to encourage mathematical thinking and problem solving.

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HOW I TEACH

HOW I TEACH INTEREST IN A MATHEMATICAL LITERACY CLASS WITH THE AID OF AN OFFLINE DIGITAL TOOL

Andri Marais and Michelle Sephton

Drostdy Technical High School and Oxford University Press South Africa

INTRODUCTION

The topic of "Interest" has been identified by teachers as one of the critical concepts learners need to master in order to understand several of the Finance topics encountered throughout the FET phase.

By introducing the concept of interest in a visual and practical way, explaining the background and demonstrating the effect of interest with everyday examples, this important foundation to the rest of the finance topics can be laid, and teachers can be confident that anything the learners did not grasp in the Senior Phase is explained before they have to apply the knowledge.

CONTENT

When introducing subjects such as Interest it is not only important to explain the background, i.e. what it is, what it means and where we find it, but also to demonstrate the key parameters and show the effect each of them has.

The key parameters in understanding interest are:

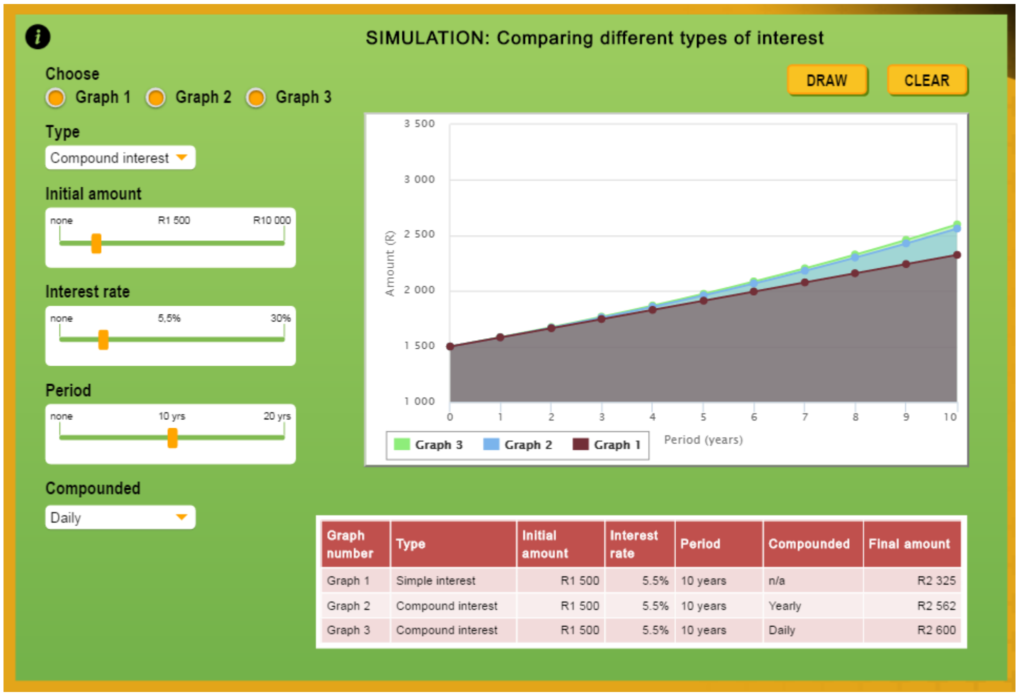
- the type of interest,
- the interest rate offered,
- the amount involved, and
- the period involved.

By using everyday examples and a new offline digital product, the effect of each of these on investments, savings and loans can be visually and dynamically investigated – allowing learners to grasp the importance and impact of these parameters.

In the presentation questions such as the following will be posed:

- Will I earn more interest if I invest a larger amount, at a higher interest rate but for a shorter time?
- How much will I save if I pay the loan off quicker?
- What is the difference in final amount if I compound the interest daily rather than yearly?

This shows the effect of type of interest and compounding periods for the same amount, interest rate and period, in answer to the question: How does type of interest and type of compounding affect my investment?



Teaching "Interest" in this way has refreshed my lessons and learners have been more engaged in the topic. They seem to grasp the concepts better and interacted more in class. They also found the immediate feedback and explanation given by the practice activities useful. I find the digital tool helps with new ideas on teaching problematic and abstract topics such as "Interest" with fresh ideas and examples. The interactive nature of this type of teaching makes it easily adaptable to different teaching methods, and the content is presented in a way that generation Z can relate to. However, it has the disadvantage in that it can become easy to rely solely on the digital content to do all the teaching. The teacher still needs to prepare for the lesson and guide the class, and also not neglect the necessary consolidating of content.

CONCLUSION

Introducing any concept visually, allowing learners to clearly see the effect of changing parameters, and letting them "play" with the idea, gives them a more solid understanding of the concept, and also brings in an element of fun and participation in the classroom.

REFERENCES

Zoom in Mathematical Literacy Grade 10, Oxford University Press, 2017

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HOW I TEACH PROBABILITY USING AN OFFLINE DIGITAL TOOL FOR MATHEMATICS IN GRADES 10-12

Janine Wilson and Fiona Heany

Oxford University Press

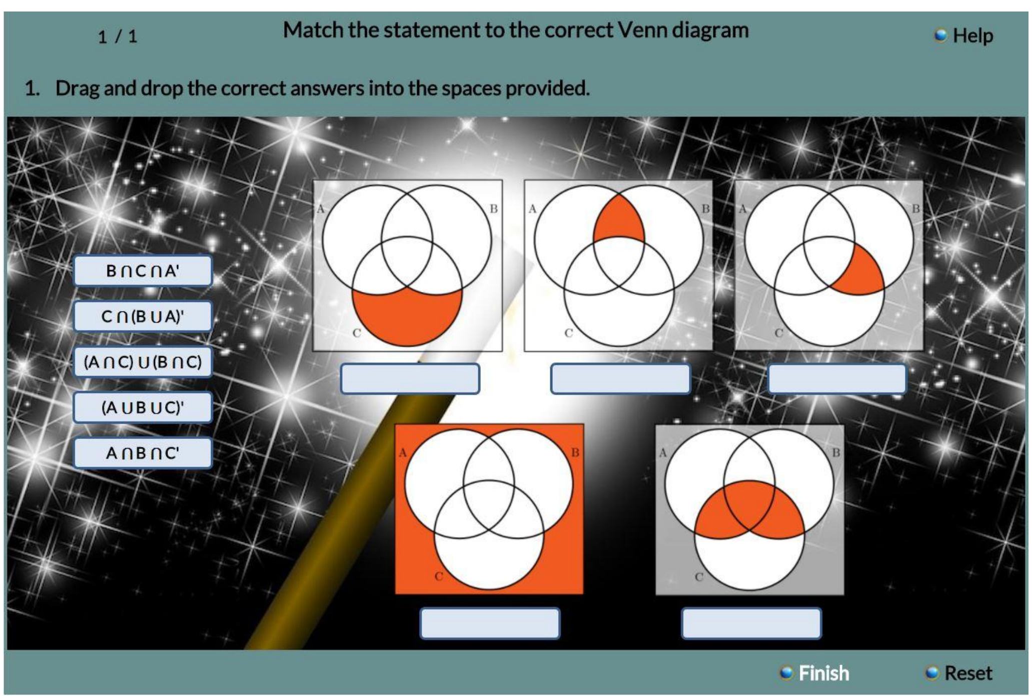
INTRODUCTION

Probability often comes up as an area of Mathematics that learners struggle to grasp. It is also part of the curriculum that is often left until late in the year (especially in Grades 10 and 12) and some teachers feel they need to rush through it to fit everything in. *Probability* does form a significant part of the marks for Paper 1 in all three grades and should be given the attention it deserves. If taught well with engaging resources, *Probability* could become a section in which learners consistently get good results.

CONTENT

I will cover the following in my model lesson:

- the probability scale
- an explanation of events A and B and how they relate
- representations of events and outcomes using Venn diagrams, contingency tables and tree diagrams
- practical examples including interactive digital activities (see below example) and exam-type questions.



Explaining a concept such as probability in a number of different ways can benefit those learners who struggle to understand the concept. Using digital tools that include video, audio and interactive elements has been shown to increase learner engagement. However, teachers should not rely on resources to do the teaching for them. They need to be available and engaged and merely use such tools as aids in their teaching.

CONCLUSION

Introducing the concept of *Probability* visually and allowing learners to work through a variety of practical examples gives them a more solid understanding of the concept, and also brings in an element of fun and participation in the classroom.

REFERENCES

Zoom in Mathematics Grade 11, Oxford University Press, 2017

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HOW I INCORPORATE TECHNOLOGY APPLICATIONS IN TEACHING THE DEAF AND BLIND LEARNERS MATHEMATICS

SC Mdlalose

Thiboloha Special School

ABSTRACT

Deaf and blind learners are struggling in mathematics, it is best that an educator is fully equipped with assistive devices such as braille machines, embosser printer, computer jaws, slate and stylus, apex and voice recorder for the blind learners. Not just having this above mentioned device but also have the skills and passion to use them effectively. Educators should act as a role model in inspiring learners to be interested in technology by using other teaching tool and challenge learners to search for more information on the internet.

INTRODUCTION

Today, more than ever, the role of educational technology in teaching and learning on learners with learning barriers especially deaf and blind learners is of great importance because technology enhances skills and cognitive characteristics. According to Dr Lazar, 2015 educational technology is classified in three categories.

1. Technology as tutor (computer gives instructions and guides the user)
2. Technology as a teaching tool (computer apps monitored by teacher)
3. Technology as learning tool (internet used as source of information)

The research done by Lowther et al. (2002) supported by Leu et al. (2009) state that all the learners in the poorer areas very rarely use the internet as a learning tool, teachers are not exposing the learners to technology as teaching tool and yet learners of the 21st century are the living legends of technology and should be exposed as they are all going to meet other learners who have advantage in learning in different high education institutions. So it is our responsibility as educators to share and expose the little resources we have with our leaners to prepare them accordingly.

These are the recommended technology apps:

1. CASIO CALCULATOR (BLIND AND DEAF LEARNERS)

2. WHATSSUP APPS (BLIND AND DEAF LEARNERS)

3. HEYMATHS (DEAF LEARNERS)

4. PHOTO MATHS (DEAF LEARNERS)
5. VOICE (BLIND LEARNERS) AND VIDEO RECORDER (DEAF LEARNERS)
6. ANDRIOD PHONE (BLIND AND DEAF LEARNERS)

In conclusion technology is playing a vital role in learners with special needs more especially blind and deaf learners as all the assistive devices are part of their daily life.

CONTENT

My blind learners are still using braille machines, stylus and slate, Apex or magnifying tools or talking calculator as assistive devices so as my deaf learners are still using hearing aids as assistive device. My lesson is all in PowerPoint which implies that I use a projector, interactive whiteboard, sound speakers and incorporating the above apps.

REFERENCES

Lowther et al. 2002 Do One-To-One Initiatives Bridge the Way To 21st Century Knowledge And Skills

Leu et al. 2009 Expanding The New Literacies Conversation

Dr Lazar. 2015 The Importance of Education Technology in Teaching, Vol 3. No 1

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HOW I TEACH FRACTIONS

Mashiya Pithi

Axium Education

INTRODUCTION

Learners struggle with fractions from the first time they meet them in the foundation phase until they work with algebraic fractions in the FET phase. In this talk I will share some of the resources and games that I use in the intermediate phase to make learners more familiar with fractions.

CONTENT

I will begin my presentation by demonstrating how I use fraction circles and fraction strips to support learners in making sense of fractions. In particular, I will demonstrate how to use the circles and the strips to help learners visualize equivalent fractions and to be able to compare fractions. I will then demonstrate how to do addition and subtraction calculations with the circles and the strips. Finally, I will share some games that I use to give learners the opportunity to practice working with the fraction of a group, rather than only having them working with the fraction of one whole.

CONCLUSION

Fractions are an important part of school mathematics and for many learners they are the first example of mathematics not making sense. By using concrete resources and games, we can help learners make sense of fractions so that doing calculations with fractions does not just become a set of rules that don‟t make sense.

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HOW TO DO PROBLEM SOLVING IN GR 4 REGARDING RATE AND DIVISION

Gina Strauss

Rouxville Primary School, Mohokare Xhariep, Free State

INTRODUCTION

I am going to talk about how I guide the gr 4 learners to solve problems in Grade 4 because all learners struggle with problem solving. Combined with this I am going to share specifically how I teach problem solving regarding division (2-digits by 1-digit) eventually leading up to the solving of rate problems. According to CAPS Gr 4 learners must be able to solve problems involving whole numbers comparing two quantities of different kinds, better known as rate. I am also going to talk about doing Curriculum Differentiation in my lesson presentation of the problem solving and rate by using the CPA (Concrete, Pictorial, Abstract) teaching approach and also in Assessment by asking questions on different levels.

I have chosen to talk about this because all learners struggle to solve word problems, also rate is a very difficult concept and educators struggle to carry this concept across to the children.

CONTENT

Introduction

Learners skip-count forwards and backwards aloud and clap in rhythm in numbers. I ask quick verbal division sums. After this I divide them into groups and ask easy verbal problem sums. Learners discuss and give the answers verbally, I give marks for correct answers and reward afterwards by giving sweets to winning groups.

Lesson presentation

I give an easy problem sum on the board and take the class through the steps of solving problems and give them the info page on the steps of problem solving to glue in to their reference books.

"You receive 12 sweets and must divide them equally between you, your two brothers and your little sister. How many sweets does each one get?"

STEPS FOR SOLVING WORD PROBLEMS

- STEP 1: Underline the key words
- STEP 2: Circle the question
- STEP 3: Ask yourself questions to make sure you understand the story
- STEP 4: Make a plan – ask questions, how are you going to solve the problem
- STEP 5: Check your plan and calculations
- STEP 6: Follow through
- STEP 7: Write a number sentence

CONCLUSION

I conclude by sharing how I do Assessment using Curriculum Differentiation: I show one example of a recording on different levels, learners must only one of the following:

The advantage of this is that all children, no matter what level will be able to do one of these sums. A is the basic sum according to CAPS, key words are underlined and a clue is given, if this is done correctly children will receive the 4 points. B is more difficult and children who can do B will receive 4 points plus a bonus. C is quite difficult for Gr 4 and children who can do this will receive the 4 points plus 2 bonus points. This encourages children to work accurately rather than receive bonus points, but also rewards children who work accurately and try something more difficult.

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CPA training 2016

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Training from Inclusive on Curriculum Differentiation and Assessment in 2017

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HOW I TEACH MATHS TO LEARNERS WITH DYSCALCULIA

Diau Ledimo

Kgato Primary School

INTRODUCTION

Dyscalculia is a learning disability that involves inability to work with numbers and lack of understanding of numbers. It is often known as dyslexia of numbers. It is a confusion of numbers that exist in the brain. Dyscalculia is not a life sentence-children can be taught math even with dyscalculia different approach need to be used. Learners with dyscalculia often struggle with the following: word problems, making change when buying at the shop, interpreting math problems, computation and transferring of information, lack of numbers sense (simple break with numbers).

CONTENT

Application of intervention strategies

Manage learners"anxiety: learners can play by counting
1,2,1,1,2,3,2,1,1,2,3,4,3,2,1,1,2,3,4,5,5,4,3,2,1

They first count slowly. Then they count as quick as they can.

Make them work on concrete material

- Equivalent fractions fitting of egg boxes.
- Adding numbers by using manipulatives.
- Addition example adding with 9 mentally.
- Example 47 + 9= 56 digit 4 say the number that comes just after 4, and on the digit 7 say the number that comes just before 7.
- Playing multiplication cards.
- For example: flashing of cards.
- The learner flashes 7 on the card and the other one flash the 2.
- Playing multiplication tables (video)

CONCLUSION

Active learning strategies engage learners instead of learner passively listening to the teacher. Element of active learning are corporative learning, peer teaching and usage of technology. Active learning is the opposite of traditional teaching where learners listen to the teacher and are never engaged.

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