ASSOCIATION FOR MATHEMATICS EDUCATION OF SOUTH AFRICA

Muckleneuk Campus, Pretoria

Gauteng

Proceedings of the 26th Annual National Congress of the Association for Mathematics Education of South Africa

Developing Equitable Mathematical Teaching and Learning Practices that Empower Teachers and Learners in the 4IR Era

14th– 16th July 2021

Muckleneuk Campus

University of South Africa

VOLUME 2

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Association for Mathematics Education of South Africa (AMESA)

P.O. Box 54, Wits, 2050, Johannesburg

Proceedings of the 26th Annual National Congress of the Association for Mathematics Education of South Africa, Volume 2, 14th-16th July 2021, Pretoria, Gauteng.

First published: July 2021

Published by AMESA

ISBN: 978-0-620-94780-0

Review process

Each of the submissions accepted for publication in this volume of the Proceedings (*Long Papers*) were subject to a blind peer review by experienced mathematics educators and academics. The academic committee considered the reviews and made a final decision on the acceptance or rejection of each submission, as well as changing the status of submissions.

Number of submissions: 62
Number of plenary paper submissions: 03
Number of long paper submissions: 34
Number of short paper submissions: 07
Number of workshop submissions: 06
Number of 'How I Teach' paper submissions: 08
Numbers of submissions rejected: 02
Number of submissions withdrawn by authors: 02

We thank the reviewers for sharing their time and expertise to reviewing the submissions

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Foreword

For educators, the employment of various technologies associated with the 4IR, is currently the subject of lively debate. The discussions centre around how to best equip learners with 21st century skills needed in an evolving market where many future jobs cannot even be imagined at present. In the African context, innovative solutions are needed to address challenges such as poverty, inequalities, famine and lack of proper housing – all critically impacting the ability of the citizens of Africa to access, afford and utilize educational technology. In order to facilitate the development of 21st century skills among learners, techno-savvy pedagogies are required to foster critical thinking, complex problem solving, effective communication and collaboration, and self-directed learning. The current congress theme, '*Developing Equitable Mathematical Teaching and Learning Practices that Empower Teachers and Learners in the 4IR Era*' was conceptualized against this background. The plenaries, presentations and workshops of the AMESA 2021 Congress further highlight and explore these themes, in addition to various approaches to optimise teaching in the mathematics classroom.

We view the COVID-19 pandemic as a quintessential adaptive and transformative challenge that affects learning from all corners of the globe. It is namely critical for learners to have access to teachers who are *au fait* with the innovative application of technology and who can equip learners with skills needed for the evolving career landscape. This is a call for the mathematics classrooms in 4IR, not only to provide sustainable education, but also to promote core values and qualities of social justice and equity for all. Subsequently, there is a need to reflect on and rethink the way in which mathematics is taught in South Africa. Thus, themes cover problem solving, empowering pre-service teachers with skills on the creation of flipped classrooms and how mathematics teaching and learning can be decolonised.

We trust that this virtual congress will inspire you to reconsider how you effect quality and equality within the teaching and learning of mathematics beyond the two and half days. You will continuously use the two volumes as a resource in your classrooms, workshops and seminars and share the knowledge gained, with other colleagues in the mathematics fraternity.

Lastly, we remain thankful to all our presenters and their reviewers of papers for their immense contribution to 26th Annual National Congress of the Association for Mathematics Education of South Africa. Keep Well and Enjoy!

Academic Coordinator

Zingiswa Jojo

July 2021

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Selecting the appropriate example set to teach mathematics

Benadette Aineamani

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Selecting and using examples in a lesson, be it spontaneous or pre-planned examples, is not a means to the end. In this paper, I present one of the main findings to a study I conducted on the role of the teacher in developing learners' mathematics discourse and understanding. With respect to Zaslavsky's argument, in the lessons observed, some content and activities had great potential to bring the intended object of learning to the fore. However, this potential was not realized. The implication of this finding is that the teacher may have to take further effort to ensure that the examples selected are pedagogically useful (Zaslavsky, 2010), and enacted in a way that allows for the usefulness to be made explicit.

Introduction

Many studies have engaged with and explored the use of examples for teaching new mathematical concepts (Knuth, Zaslavsky & Ellis, 2019; Zaslavsky, 2019; Huang, 2017; Rissland, 1991; Mason & Pimm, 1984). However, there are not many studies that have investigated the selection of content and activities, with a focus on how the content and activities selected play a role in helping the teacher to develop learners' mathematics discourse and understanding. It is through examples that concepts are illustrated, demonstrated, and explained to learners. As Rissland (1991) states, teaching mathematics requires careful selection of examples. Zaslavsky (2010) emphasises the role of examples, and he states that "examples are essential for generalization, abstraction, and analogical reasoning" (p. 107). Zodik and Zaslavsky (2008) argue that the examples selected by the teacher may hinder or enable learning. It therefore becomes imperative to select appropriate examples. The content and activities[1] that the teacher selects in the process of developing learners' mathematics discourse and understanding are crucial. However, research on developing learners' mathematics discourse and understanding has not given the content and activity selection the attention that it requires. Therefore, content and activity selection were one of the focus areas in my study, and this is the finding that I present in this paper.

Theoretical framework

For the study, I drew on two frameworks: Mortimer and Scott (2003) framework for meaning making in a dialogic classroom, and variation theory. Vygotsky's theory was used by Mortimer and Scott to develop their framework. The main argument that was drawn on by Mortimer and Scott is that learning originates from social situations and that ideas are rehearsed between people using actions such as gestures, talk, visual images and writing. Mortimer and Scott's framework were suitable for my study because of the focus on talk and learning through meaning making within the science discourse. Mortimer and Scott's (2003) framework provided me with 'aspects of analysis' such as patterns of discourse and communicative approach that explain how the teacher plays the role of making the scientific story available to learners. The other part of the theoretical framework was Variation theory, and this informed the aspect of content and activity selection. Choosing content and activities for teaching is important. Huang (2017) argues that learners' understanding of any mathematics ideas is dependent on the examples that are selected by the teacher. This was confirmed by the data which I collected for the study. In my study, all the lessons observed were centred around the content and activities that the teachers selected. I refer to these as the example set of a lesson. Selecting an appropriate example set is an art in mathematics teaching (Michener, 1978; Huang, 2017). There are four patterns of variation: Contract, generalisation, separation and fusion, and I drew on these patterns to unpack the content and activity selection in the study.

Contrast refers to the comparison between the critical feature of the object of learning and the other aspects that are not critical to the object of learning (Marton, Runesson, & Tsui, 2004). For example, in the context of a linear function, $y = 2x + 1$ can be contrasted with $y = 2x^2 + 1$ in order to discern the critical features of a linear function. *Generalisation* refers to experiencing features of the object of learning that are varying in order to discern the irrelevant features. For example, in the context of linear functions, learners may be given different types of equations, but all equations of linear functions: $y = x + 1$; $y = 2x + 1$; $y = 3x + 1$. The three equations given show the invariant power of x, a critical feature of an equation to be referred to an equation. *Separation* refers to when certain features of the object of learning are kept invariant while varying the others. Separation occurs through controlled variation, where certain aspects are kept invariant while varying others (Watson & Mason, 2006; Marton et al., 2004). In the context of a linear function, to draw attention to the power of x, learners may be given two equations $y = 2x +$

1 and $y = 2x^2 + 1$. Allowing the power of x to change draws the learners' attention to it, and this is a critical feature of the equation of a linear function. *Fusion* occurs through synchronic simultaneity (Marton et al., 2004). Synchronic simultaneity refers to the ability of discerning more than one critical feature of the object of learning in a given example (Runesson, 2005).

Selecting appropriate examples to teach mathematics concepts

One main aspect that teachers need to consider is that an example may exemplify various mathematical ideas (Antonini, Presmeg & Mariotti, 2011). Examples enable the teacher to provide learners with opportunities to engage with essential parts of teaching mathematics such as analogical thinking, generalisation, conceptualisation, argumentation, and abstraction (Yanuarto, 2016). Watson and Mason (2006) argue that an example is any item that is derived from a larger aspect, which can be used by learners to reason and come to a generalisation about a concept. Peled and Zaslavsky (1997) argue that while many examples may be used to illustrate a concept, some examples have more powerful pedagogical aspects than others. For example, when representing a linear function, giving an example such as $y = 1$ and $y = 2$ may not be as effective as giving learners the examples $y = 2x + 1$; $y = 3x + 1$ and $y = 2x$, to engage with the critical features such as the co-efficient and the y-intercept. In the example of equations above, the learners need to be able to see the general equation ($y = mx + c$) of a linear function, and also be able to identify the coefficient and y-intercept in any particular equation, by drawing on the general equation.

In order for an example set to be selected appropriately, learners' prior knowledge needs to be in focus because learners come to any learning environment with prior knowledge on the skills and concepts that are presented to them (Bransford, Brown & Cocking, 1999; Campbell & Campbell, 2009). As Bransford et al. argue, the prior knowledge that learners bring has a considerable effect on what they 'see' and how they analyse new concepts and develop skills in the classroom.

Zodik and Zaslavsky (2008) identified two types of examples: pre-planned and spontaneous examples. Pre-planned examples are carefully planned and used as part of a planned lesson, while spontaneous examples are those examples which the teacher chooses in-the-moment, when faced with a completely new situation in the classroom. Zodik and Zaslavsky argue that if there is some evidence that the teacher included an example in the planning process of the lesson, then it is pre-planned. Zodik and Zaslavsky argue that are two reasons for choosing spontaneous examples: 1) to address learners' responses, especially when learners gave incorrect responses, and 2) response to limitations of pre-planned examples during the lesson. As shown by Zodik and Zaslavsky's study, choosing examples, whether pre-planned or spontaneous examples is not an easy task.

Everyday contexts in examples selected is another aspect that is important to discuss in the process of selecting an example set. Hedegaard and Chaiklin (2005) discuss a 'double move' as a process where everyday ideas are used to develop scientific ideas, and scientific ideas are drawn on to clarify everyday ideas. Hedegaard and Chaiklin argue that for learning to be meaningful and powerful, teachers need to keep in awareness that everyday ideas and scientific ideas are important and that they complement each other. The examples with everyday context may be spontaneous or pre-planned. Jo (1993) argues that there has been a debate that suggests that including everyday context helps learners to learn mathematics better and may also help learners to relate abstract mathematics to real world problems. This debate has led to inclusion of everyday context into the teaching of mathematics at various levels, from materials to assessments and it is also evident in classroom teaching (Jo, 1993). The relation between including of everyday context has three main advantages: 1) recognise the requirements of the activity, 2) retrieve information from previous knowledge about the context that learners can relate to and 3) translate the information provided to suit the demands of the activity (Jo, 1993). If the three things do not happen, no matter how powerful the context, misconceptions may occur. One difficulty of everyday context that Jo highlights is the 'choice' of context to be used, because learners may have different contexts that they can relate to.

Research design and methodology

I conducted a qualitative study. Brantlinger, Jimenez, Klingner, Pugach and Richardson (2005), define a qualitative research design as a "systematic approach to understanding qualities, or the essential nature, of a phenomenon within a particular research" (p. 195). Qualitative research was appropriate for my study because it allows the researcher to view reality as multilayered and interactive (Schumacher & McMillan, 1993).

To collect rich data for my study, I used a 3-phase approach. The sample selection was opportunistic because I had access to teachers who were part of a professional development course that was run by the University. The first phase of data collection was pre-observation teacher interviews, the second phase was classroom observations and the third phase were post-observation teacher interviews.

For this study, Cohen, Manion and Morrison's (2000) purposive sampling was used. The schools and the teachers who took part in phase one were selected from a group of available schools, but the participants for phase two and three in my study were selected purposefully. The teaching experience of the teachers and the qualification level did not vary in this study. I selected Grade 10, with a focus on functions as a topic. Functions is a crucial topic in mathematics, and Grade 10 learners are introduced to the topic for the first time. Phase one comprised six teachers, phase two and three comprised three teachers, selected from the six teachers in phase 1. Phase 2 and 3 had two teachers from the same school and 1 teacher from another school. The teachers were teaching the concept of functions to Grade 10 learners. I referred to the teachers in my study as Teacher A, B and C. I observed 11 lessons in total.

Drawing on the theoretical framework and literature review, I developed an analytical framework for study as shown in the figure below.

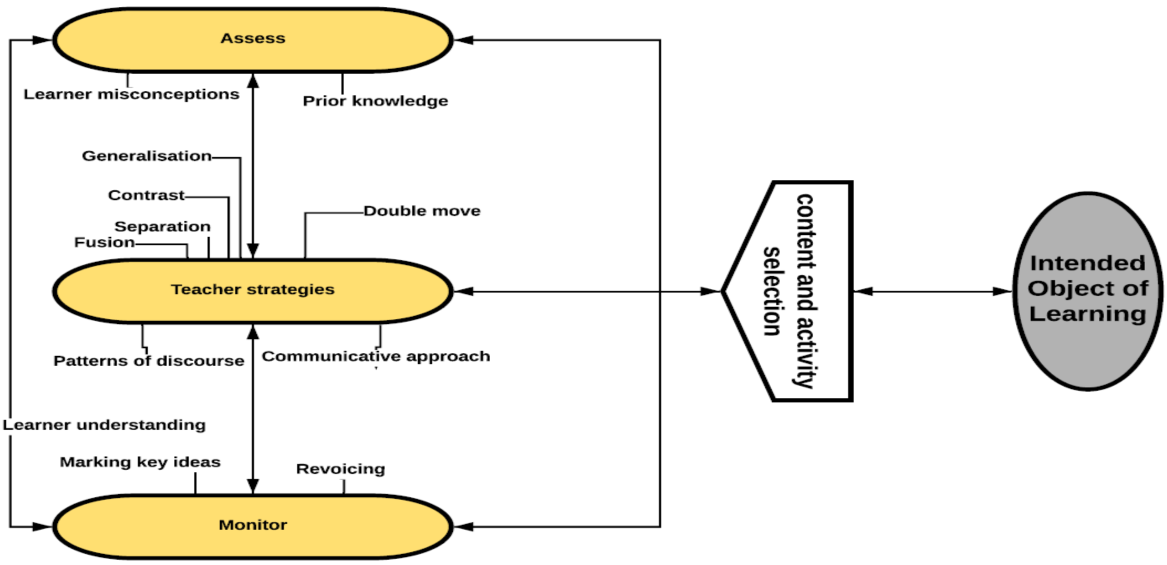

Figure 1: Analytical framework of the study

The intended object of learning was the starting point of the analysis, followed by the content and activity selection. The arrow between the intended object of learning and the content and activity illustrates that there is a back and forth movement between the two. The teacher may move back to confirm the intended object of learning after content and activities have been selected.

Findings and discussion

Teachers A, B and C all selected different kinds of content and activities to enact the intended object of learning in all the lessons observed. The examples selected by the three teachers were either spontaneous or pre-planned. Spontaneous examples were selected 'in-the-moment' as defined by Zodik and Zaslavsky (2008), while pre-planned examples were selected before the lesson. The teachers drew on scientific and everyday contexts.

The kind of content and activities that the teachers selected, and how they were enacted, did not provide opportunities for learners to explain and justify their responses in the lessons and hence develop mathematics discourse. The teachers were aware of the role of examples in teaching and learning; however, the teacher-learner engagement was not geared towards fostering a comprehensive mathematics discourse. According to Bills, Dreyfus, Mason, Tsamir, Watson and Zaslavsky (2006), there are two main characteristics that make an example pedagogically useful. The first is transparency of the example and the second is the ability for examples to be generic and hence foster generality.

By transparency, Bill et al. (2006) refer to an example that has features that make it exemplary to the target audience. One way of making the example exemplary is using everyday context that learners can relate to. Zaslavsky and Peled (1996) argue that unless learners are exposed to a rich example set, they may be attracted to the noise in the examples given. Another way is ensuring that prior knowledge is considered when selecting the example (Campbell & Campbell, 2009). Watson and Mason (2006) concur with Zaslavsky and Peled (1996), and Campbell and Campbell (2009), and they argue that examples need to be extensive for connections to be made. It is worthwhile to provide learners with several examples from which to get an overall sense of what is being taught and hence generalisation. The findings from my study are summarised in Figure 2 below.

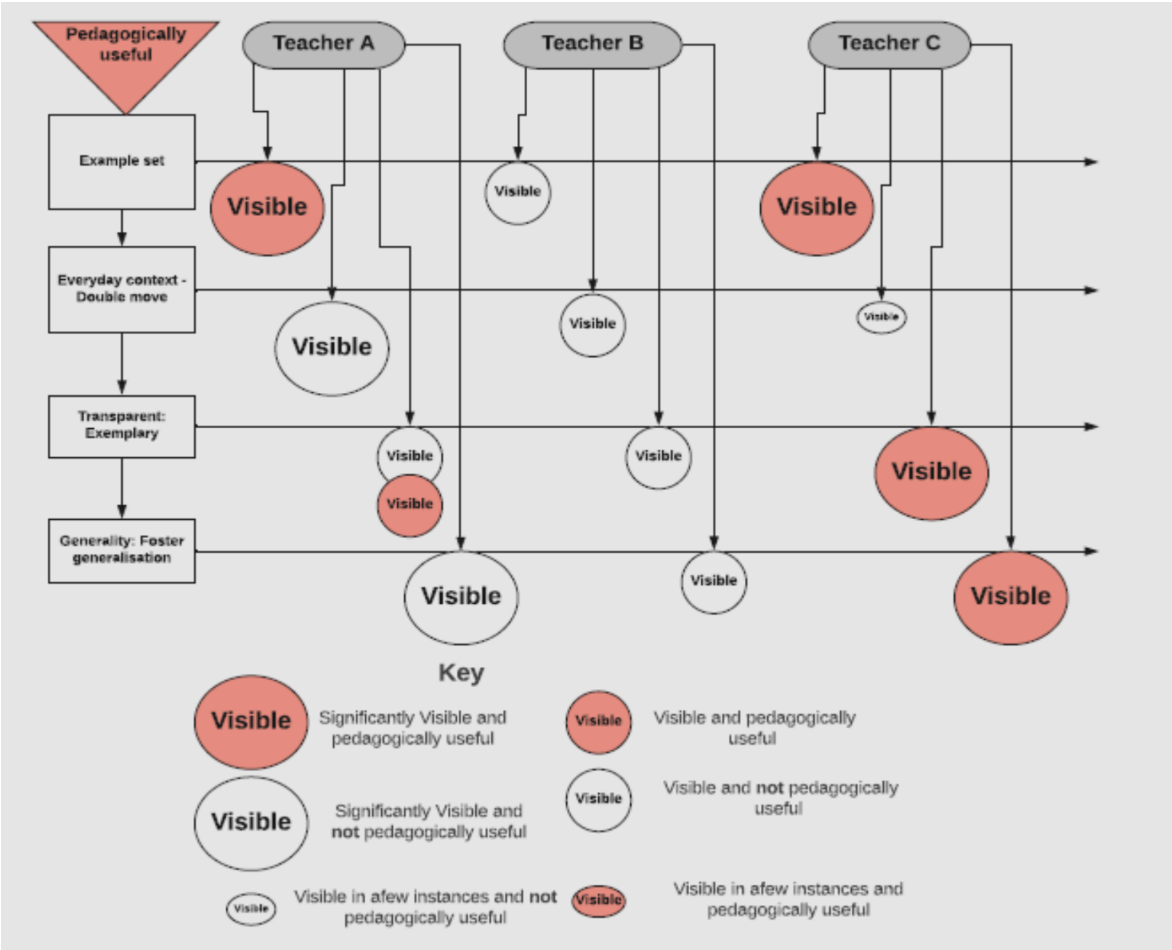

Figure 2: Teacher A, B and C's example set

Figure 2 above is a summary of how pedagogically useful the teachers' example sets were in developing learners' mathematics discourse and understanding, with a focus on functions. Within the teachers' example sets, there were content and activities that attempted a double move, Teacher A's double move being the most visible and Teacher C's the least visible. However, all the three teachers' double moves were not pedagogically useful. With regards to the example set being transparent and being exemplary to the target audience, Teacher A and B's example sets were not transparent as seen in the content and activities presented to the learners. For example, Teacher A selected an activity of a factory in relation to the concept of a function and this was not exemplary and hence not transparent, and therefore it was not pedagogically useful. Some of Teacher A's content and activities were transparent and learners were able to relate to them, hence the two circles in Figure 2 under Teacher A. Teacher C selected an example set that was transparent in relation to what she was teaching on the concept of functions. However, transparency for Teacher C did not mean drawing on the everyday context to make the examples exemplary, she drew on the scientific context in her example set. On the possibility to foster generality, Teacher A and C's example sets had possibilities for generality, with Teacher C's example set being pedagogically useful. Teacher B's example set did not foster generality. Yes, it is the teacher's role to ensure that the example set selected is as transparent as possible, by drawing on content and activities that are exemplary to the target group and in relation to the intended object of learning. However, it is not an easy task due to the complex nature of selecting an appropriate example set.

Conclusion

In conclusion, the study highlighted the need to re-emphasis of the notion of example set in the teaching and learning of mathematics, especially in critical topics such as functions, to raise teacher awareness on how to develop an example set before and during the lesson, and what should be considered when developing the example set. Curriculum documents and the role that the teacher plays in interpreting and making decisions about how to implement the intended object of learning needs to be emphasised. Strategies such as double move, patterns of variation, and assessment need to be emphasised and linked to how teachers play a vital role in selecting and enacting the example set to develop leaners' mathematics discourse and understanding.

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A comparison of fourth-year student teachers' cognitive demands and knowledge domains in selected mathematics topics: Implications for teacher education

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The purpose of the Primary Teacher Education Project (PrimTEd) was to assess the fourth-year student teachers' performance on the five content knowledge domains: whole numbers, geometry, rational numbers, algebra, and measurement. A PrimTEd test was administered online to 232 fourth-year student teachers, 30 student teachers from university A, and 202 fourth-year student teachers from three universities that formed a pilot group in the final year of their studies (Month 2018). All groups of fourth-year student teachers voluntarily participated in the PrimTEd test. The findings reveal that students from university A performed better than the 2018 pilot group in all the three cognitive demands, namely, lower, higher, and pedagogy, with more than 15 percent in lower and higher and 3 percent on pedagogy. The results of this study have implications for initial teacher education, specifically the way cognitive demands and knowledge domains are introduced and approached during the preparation of fourth-year student teachers.

Introduction

Efforts have been made to develop the student-teacher’ knowledge of the content domains and cognitive demands in mathematics tasks. The literature on cognitive demands reveals that fourth-year student teachers have difficulty distinguishing between the highest cognitive demand procedures with connections and doing mathematics. Some of the studies conducted in this area focused on developing fourth-year student teachers' knowledge of cognitive demands as they ask learners questions during teaching practice (Lin & Tsai, 2015). Anthony, Walton, and Viviani (2020) examine how preservice teachers conceptualize cognitive demands when selecting a task. Their findings revealed that preservice teachers tend to operationalize the cognitive demands to support learners' disposition and ability to determine the problem difficulty.

Most of the literature on cognitive demands and mathematical tasks focuses on the different phases (in South Africa Foundation Phase, Intermediate Phase, Senior Phase, and FET Phase) at the school level. For instance, some of the studies have been conducted by Henningsen and Stein (1997), Gogovska (2014), Berger, Bowie, and Nyaumwe (2010), Antonijevic (2016), Georgius (2014), Kessler, Stein, and Schunn (2015), and Strayer and Brown (2012). In general, these studies have explored the various levels at which the learners in schools can cope and how they focus on different cognitive levels.

One of the studies that focused on student teachers (preservice teachers) was conducted by Cheng, Feldman, and Chapin (2011). This study is similar to our study in several ways. Firstly, it focused on an elementary level which to a large extent relates to the Intermediate Phase in which our study was conducted. The study by Cheng, Feldman, and Chapin also focused on geometric measurement, the mathematics content close to the content domain of geometry that we used in our study. The study by Cheng, Feldman, and Chapin, used discussions in small groups to investigate preservice teachers' reasoning. The study showed that the high level of cognitive demand is maintained when the preservice teachers focus on providing explanations, justifying procedures, and generalizing. Preservice teachers participating in the study consistently asked each other (in their small groups) for explanations and were often dissatisfied with correct solutions that they did not understand clearly.

Our view is that content domains are essential to determine the level of cognitive demand of tasks of fourth-year student teachers, however, through observation of fourth-year student teachers' selection of tasks, they identify low-level tasks as high-level tasks, such as acquiring manipulatives and real-world context (Lin & Tsai, 2015).

This paper reports on a study that compared the performance of the fourth-year student teachers in the pilot group of the PrimTEd Project with fourth-year student teachers at university A on the cognitive demands and the content domains of assessment test on the mathematics content domains for grades 4to7. PrimTEd is part of the Teaching, and Learning Development Capacity Improvement Programme (TLDCIP) developed and implemented through a partnership between the Department of Higher Education and Training and the European Union. PrimTEd comprises academics in Initial Teacher Education (ITE) who came together to develop a common core set of standards for primary school mathematics teachers, develop and administer standard assessment instruments, and share learning materials and approaches to work-integrated learning. In the comparison reported in this paper, the authors used the PrimTEd online mathematics test developed by members of the Assessment Team of the PrimTEd Project to test the fourth-year student teachers' performance on the cognitive demands and the content domains.

Null Hypothesis

- There is no significant difference in the performance on content domains between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands.
- There is no significant difference in the cognitive demands between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands.

Methodology

A quantitative research method was used to investigate fourth-year Intermediate Phase mathematics student teachers' performance in the five content domains and the three cognitive demands that formed part of the online PrimTEd test and developed by the PrimTEd Team. The PrimTEd Team administered the PrimTEd online test to 202 fourth-year student teachers to three universities who formed a pilot group in 2018. In addition, the authors administered the PrimTEd online test to 30 fourth-year intermediate phase student teachers in University A in 2018. Thus, a PrimTEd online test allowed the authors to examine the significant difference in the content domains between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands.

The content domains were spread across the five topics as follows:

- Whole numbers and operations: 24%, 12 questions out of 50
- Rational numbers and operations: 38%, 19 questions…
- Patterns, functions, and algebra: 16%, 8 questions.
- Geometry: 8%, 4 questions
- Measurement: 14%. 7 questions

Results

The descriptive and inferential statistics were used to test the first and second null hypotheses about the fourth-year student teachers' cognitive demands and the content domains in two universities. So we will start by testing the null hypotheses on the content domains and then later the cognitive demands.

H_1 There is no significant difference in the performance on content domains between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands. Therefore, the null hypothesis is interpreted in the table below.

Table 1. Performance of fourth-year student teachers per content topic

Content Area	**University A Mean**	**Pilot group national mean**
Whole numbers	64.95	53.39
Geometry	70.83	60.32
Rational numbers	61.45	41.03
Algebra	57.92	44.15
Measurement	74.76	56.06
P-value	0.211400176	

From the PrimTEd online test scores, the mean scores for university A and the pilot group compared to all the five content domain topics. The fourth-year student teachers in university A scored higher than their counterparts in the pilot group in all the five content domains. For example, the University A fourth-year students teachers scored from the highest to lowest, measurement, geometry, whole numbers, rational numbers, and algebra. In contrast, the pilot group scored from highest to lowest geometry, measurement, whole numbers, algebra, and rational numbers. The t-test was conducted to test the significant difference in performance in all the five content domains between universities A and the pilot group. The p-value of 0.211400176 supports that there is no significant difference between the performance of University A and the pilot group. The difference in performance may have happened by chance.

H_2 There is no significant difference in the cognitive demands between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands. Therefore, the null hypothesis is interpreted in the table below.

Table 2. Performance of fourth-year student teachers per cognitive level

Cognitive Demand	**University A mean percent**	**Pilot group national mean**
Lower	77.36	58.67
Higher	55.59	40.14
Pedagogy	35.33	29.97
p-value	0.006755732	

From table 2, it can be said that university A showed higher mean scores in all the three cognitive demands lower, higher, and pedagogy with 10 percent and more. Furthermore, University A obtained the highest mean score in the cognitive level higher, while the pilot group obtained the lowest mean score. This suggests that the fourth-year student teachers possessed a higher level of cognitive domains than their counterparts in the pilot group. We conducted a t-test and the p-value of 0.006755732, $p \leq 0.5$, showing no significant difference in university A and the pilot group performances on cognitive demands.

Discussions

The purpose of the study was two-fold, examine whether there is no significant difference in the cognitive demands between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands. Firstly, the findings supported the null hypothesis, which revealed no significant difference in the cognitive demands between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands. In contrast, Fonseca et al. (2018) revealed that the pilot group performed similarly to the comparing cohort in the fourth-year intermediate phase student teachers' performance in both the cognitive demands and the content domains the data collected. Therefore, this study's results do not necessarily imply that fourth-year student teachers at University A are better than those of the three universities that participated in the pilot study, and the difference may have happened by chance.

Secondly, the study tested whether there is no significant difference in cognitive domains between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands. The findings revealed a significant difference in the cognitive demands between fourth-year student teachers in University A and three universities that form a pilot group on cognitive demands. Other variables helped university A fourth-year student teachers perform better than their counterparts, though the variables have not been identified. The results show that the fourth-year student teachers in University A performed better in the Lower Cognitive Demands than the Higher Cognitive Demands. The findings have significant implications as it suggests that the fourth-year student teachers in the pilot group seem not prepared for higher cognitive demands. The fourth-year students' finding suggests that the student teachers at University A may have been prepared for the PrimTed online test due to the course material covered in their classes before the test was administered to them. One of the implications for preparing fourth-year student teachers in content domains is that Whole Numbers, Geometry, Rational Numbers, Algebra, and Measurement are essential in teacher education programmes.

Thus, when the selected universities prepare fourth-year student teachers to teach mathematics in the Intermediate Phases, the facilitators need to identify and emphasize improving fourth-year student teachers' ability to deal with questions that focus on higher cognitive demands and not just be comfortable that such student teachers can perform well on lower cognitive demands.

Conclusion

Even though the performance of University A in the five content areas of Whole Numbers, Geometry, Rational Numbers, Algebra, and Measurement is higher than that of the Pilot Study, the content domains still require greater emphasis in the Teacher Education Programme. These content domains were meant to serve as building blocks for related content domains at the Senior and FET Phases. Therefore a better understanding by the fourth-year student teachers would translate into more appropriate strategies when these are taught to the learners in the Intermediate Phases.

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ENGINEERING STUDENTS' UNIVERSITY ENTRY LEVEL MATHEMATICS: REFLECTIONS ON ALGEBRA

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Students enter university with different educational needs based on their past experiences with mathematics. Diagnostic tests allow us to understand the mathematics levels of our incoming students and learn where more support may be needed. This is particularly relevant given the possible disparity in schooling due to the 2020 pandemic lockdowns. Vygotsky's social constructivism is the theoretical lens which frames this early-stage action research. Participants were first-year engineering students at the University of Cape Town. Gradescope was used to mark 205 responses to a diagnostic pre-calculus mathematics test and a statistical analysis of the responses was conducted. Lowest performing questions on a diagnostic test involved simplifying fractions, sketching a rectangular region from inequalities, finding domain by solving a quadratic inequality, and graphing a shifted hyperbola. The findings confirm some known errors, such as showing algebraic relationships on graphs, and present valuable insights that can inform mathematics teaching and learning at school and university.

Introduction

Students' mathematics readiness to undertake and succeed in mathematics courses at higher education remains a topic of concern (Jacobs & Pretorius, 2016; Sadler & Sonnert, 2018; Cuevas et al., 2018). The 2021 university student intake may find that the regular difficulties common to students adapting to university are compounded by the difficulties of learning during the pandemic.

Participants were first-year engineering mathematics students who started at the University of Cape Town in 2021. Due to complications caused by the 2020-2021 lockdowns, students were not required to write National Benchmark Tests, reducing our background knowledge of these new students. The rationale for embarking on this research was our reflection on the questions: What is our readiness to facilitate the mathematics learning of the 2021 intake of students and, how can we best facilitate this learning?

The theoretical lens which frames this research is Vygotsky's social constructivism where the central belief is that students actively construct their learning based on their existing knowledge. An aspect of this theory that is particularly important in relation to identifying students' entry level competencies is the Zone of Proximal

Development (ZPD). The ZDP is theorised as the "the distance between the actual development level, as determined by independent problem solving and level of potential development, and through problem-solving under adult guidance, or in collaboration with more capable peers" (Vygotsky, 1978: 86). The findings of this research will inform the nature of scaffolding required to lead students through the ZPD as they participate in engineering mathematics.

This paper reports on a first step in action research, with cyclical stages of observing, reflecting, planning and acting. We followed the action research principle of involving multiple-stakeholders: our research team are convenors for semester courses in first- and second-year mathematics and first year physics for engineering students. The research was framed with a "democratic impulse" (Koshy, Koshy & Waterman, 2010: 10) by asking the incoming 2021 students to help us understand their familiarity with high school mathematics and inform our work towards adjustments to teaching and learning practices.

Following ethical clearance, 205 first-year engineering students wrote a mathematics diagnostic test on key high school mathematics topics relevant to engineering mathematics. To help preserve the confidentiality of the test questions, the test was written on paper in face-to-face tutorials at the start of the academic programme. Scanned papers were marked online by the researchers on the grading tool Gradescope, using error descriptions in the create-as-you-go rubric.

In this paper, we analyse four of the lowest performance questions. The questions and scores from Gradescope are given in Table 1, along with the overall performance on the test.

TABLE 1: Performance statistics of selected questions from the diagnostic test

	Max (%)	Mean (%)	Std dev (%)
Simplify a stacked fraction with a difference of fractions in numerator and denominator	100	54.4	44.1
Inequalities: Sketch the 2D region defined by intervals for x and y	100	14.6	35.4
Inequalities: Find the domain of a function of the form $y = \sqrt{x^2 - a^2}$	100	40.5	49.2
Sketch a vertically shifted hyperbola	100	45.9	50.0
Overall results	92.1	54.9	16.1

The analysis of each question follows, focusing on misconceptions and misunderstanding and the likely effect this may have on first year Calculus.

Simplifying a stacked fraction

The complex fraction in this question is what some will refer to as a "stacked fraction," with a difference of fractions in both the numerator and the denominator. The generally accepted method of simplifying these stacked algebraic fractions involves finding the respective common denominators for the numerator and the denominator and expressing each as a single fraction. The simplified numerator and denominator can then be combined into a single fraction by multiplying the numerator by the reciprocal of the denominator ("flipping and multiplying"). Some factors in the numerator and denominator may cancel with each other. Further simplification required in this question was factorising using a difference of two squares in the numerator resulting in a common factor in both numerator and denominator. Forty three percent (43%) of the students answered the complex fraction question correctly and 18% of students got close to correct answers with errors on signs before simplification. The errors with signs show an inability to match factors like (x - y) and (y - x) by factoring out -1. For those students who didn't get the correct answers there were several mathematical errors such as with the simplification of the factorisation of the difference of two squares (10%), error in simplification of two fractions subtracting each other (8%), using multiplication instead of factorisation (4%), incomplete answers (3%), and not simplifying (3%), as shown in Table 2.

TABLE 2: Student response to simplifying a complex fraction

Response	**Number of students**
Correct	89
Correct apart from signs	37
Numerator not factorised	21
Simplification error	18
Blank	13
Multiplying instead of factorising	9
Incomplete answer	8
Not simplified	8

Students' under-preparedness with numeracy demands have been reported in science and engineering (Coetzee & Mammen, 2017), nursing and health sciences (Jukes & Gilchrist, 2006) and law (Gabaldon, 2014). In addition, Coetzee and

Mammen, (2017) identified science and engineering students' prior mathematics knowledge at university entry level and found that students faced challenges with conceptual understanding of fractions. Simplifying complex algebraic fractions is part of the fundamentals of precalculus. Fraction simplification skills are used in differential calculus when finding derivatives from first principles and in limits when direct substitution fails and simplifying a complex fraction using factorising is needed before substitution can take place. An understanding of fractions is needed in integral calculus, in the partial fraction decomposition of an integrand. Differential equations may involve fractions that need to be simplified and factored before it is possible to separate the variables.

First-year university lecturers need to offer remedial action to help students to overcome the challenges faced with fractions (Coetzee & Mammen, 2017). We suggest encouraging students to go through fraction concepts using Khan Academy and including complex fraction problems as pre-work for tutorial sessions related to specific topics like solving derivatives using first principles.

Inequalities

Mathematical inequalities are useful in many disciplines such as science, engineering, and economics, and difficulties with the concept of inequalities may have far-reaching consequences in students' further studies and future work. Halmaghi (2011: 3) highlights this importance: "Inequalities are a means of expressing constraints and solving linear programming or optimization problems, and they provide a way of expressing the domain of a function, of solving limits, or of setting up research questions that relate equations to special cases."

Two low-performance test questions involve solving inequalities, the first asking for a sketch of a 2D region and the second asking for the domain of a function. The question on the 2D region presented students with two inequalities of the form $a \leq x \leq b$ and $c \leq y \leq d$. The question on the domain required students to solve a quadratic inequality since the function given was that of a quadratic expression under a square root. The average scores for the question on the 2D region and the question on domain were 16% and 40% respectively, both below the overall test average score of 56%. Table 3 shows the kinds of errors made and the corresponding tallies.

TABLE 3: Student responses to sketching a 2D region based on two inequalities

Number of students	119	31	30	13	9	3
Response	Correct shape but no shading	Blank	Correct	Horizontal line as answer	Shape as circle/oval/ parabola/line	Diamond shape

An alarming 85% of the students were not able to sketch and shade a rectangular region. We expected that in high school students would be exposed to solving inequalities, and to sketching points, lines and regions on the Cartesian plane as prescribed by grade 10-12 CAPS guidelines (Department of Basic Education, 2011). We note that many drew the correct region and failed to shade in the appropriate solution. It may be that students thought simply drawing the region was sufficient and did not realise that shading was necessary to indicate the solution, as the question did not mention the word 'shade'. Perhaps the integration of more than one idea seems to have confused them. If this is so, it is an example of how it is detrimental to mathematics understanding to learn in silos. Concepts such as the Cartesian plane, interval notation, and sketching the overlap of regions associated with restrictions are perhaps misunderstood or lacking, or students struggle to integrate these concepts.

A well-known method of teaching how to arrive at the region determined by linear inequalities is to draw the lines, indicate with arrows the side of the line that satisfies each inequality, decide whether the inequalities include the boundary lines and then shade as directed by the inequalities. A graphical approach to teaching inequalities is recommended to help students to interpret the solution through a visual image of the solution (Ndlovu & Ndlovu, 2020; Tsamir & Almog, 2001; Ward, 2016; Dreyfus & Eisenberg, 1985).

The second inequality question we chose to analyse required knowledge of square root functions and indirectly led to solving an inequality. The domain of a function is a subset for which the function is defined. In the case of this function, involving a square root not in a denominator, the expression under the square root must be non-negative, i.e. positive or zero. When the expression under the square root is a quadratic expression, students find it more complex to solve the inequality than they do with a linear inequality. In this question 60% of students did not give a correct response. There may have been a fundamental misunderstanding or lack of understanding of the concept of domain. If a student does not understand the concept of domain, they may not proceed to the inequality that will solve for the relevant subset. Many missed the fact that the expression under the square root could be zero. Others solved as if the expression was linear and gave the answer as points. The types and tallies of errors are given in Table 4.

TABLE 4: Student responses to finding domain of a function like $y = \sqrt{x^2 - a^2}$

Number of students	83	72	27	12	11
Response	Correct	Completely incorrect, e.g. $x \in$ R	Correct but incomplete domain	Correct but missing equal signs	Blank

In the progression of concepts, linear equations precede linear inequalities. The methods of solving linear equations appeared to have confused some students into thinking that the same applies for inequalities. This is an indication that newly acquired knowledge competes with intuitive beliefs, confirmed by Fischbein (1987) who notes that the structural similarities between equations and inequalities lead to incorrect but strong feelings that draw students to using similar strategies inappropriately. Fischbein (1987) advises teaching the differences and similarities between equalities and inequalities, as well as including graphical illustrations of inequalities to help students visualise the differences and similarities of equalities and inequalities.

In addition, there may have been a misunderstanding of what the solution represented, not knowing that the relevance of the solution to the function could be checked. A common method in solving quadratic inequalities is to construct a sign table with critical points where the expression equals zero and then by substituting points in the relevant intervals test to see what the sign of the expression is on that interval and at the endpoints. McLaurin (1985) and Dobbs and Peterson (1991) argue that the use of a sign chart is the best way to teach quadratic inequalities since students will have this skill when they are confronted by any other difficult inequality for which no other method of solution is available. In our inequality, a faster method involved a quick plot of the quadratic function. Piez and Voxman (2020) make a case for students learning multiple methods of solution and representations since this will hold them in good stead when solving other inequalities and will develop their problem-solving ability. Tsamir and Reshef (2006: 6) also agree with "present[ing] students with multiple methods when teaching quadratic inequalities."

The study of calculus requires an understanding of regions. In first-year engineering mathematics, students encounter areas bounded by two or more functions, and volumes of revolutions which involves rotating a region. Understanding regions will also be required in multivariable calculus and in complex numbers. The following examples show how inequalities can be used to define regions in university mathematics questions.

(a) Sketch the area enclosed by the following region S. Find the area of the region if it is finite. $S = \{(x, y) \in \mathbb{R}^2 : x \leq 2, 0 \leq y \leq e^{-x/2}\}$
(b) Sketch the region described by $\{z \in C: 0 \leq Re(z) < 1, Im(z) \geq -3\}$ in the complex plane.

The very definition of a function includes its domain, a lack of understanding here will be detrimental. Not every function is defined everywhere, and to be aware of where it is defined, students are required to find the domain. Inequalities are also used to determine intervals where a function is increasing or decreasing, and intervals of concavity.

The inequality concept map given by Halmaghi (2011) suggests a comprehensive method of presenting the concept of inequalities to students integrating the various aspects associated with it. It may be most appropriate to expose students to different ways of representing and solving inequalities. We suggest, in addition, that whenever inequalities are encountered in any topics in the mathematics syllabus, an awareness of the inequalities should be raised with students so that they may understand the uses, relevance and solution of inequalities.

Sketching a vertically shifted hyperbola

Transforming a function from a basic function and sketching the graph of the new function from the old function is useful in visualising the behaviour of a wide sample of functions. Visualising transformed functions has consequences for university mathematics and beyond, for example in modelling, engineering and economics.

Incoming university students are required to have some knowledge of functions. Algebraic manipulations with functions are relatively easy for students to master (or rote learn) but many students have trouble with sketching the graph of a function. Students' reluctance to sketch could possibly be linked to not understanding that y is dependent on x and that y changes as x changes.

The sketching question we analysed required the student to sketch a graph of the function $y = 1+1/x$. To do so, students needed to know the shape of the basic hyperbola $y = 1/x$. In addition, students needed to understand how to obtain a new function from a basic hyperbola that required a vertical shift. The horizontal asymptote shifted one unit up to become $y = 1$ while the vertical asymptote remained at $x = 0$.

The responses are detailed in Table 5. Fewer than half of respondents (45%) answered correctly, suggesting insufficient or inadequate practice with graph sketching. The high percentage (33%) of blank responses may be caused by not recalling the shape of the graph for $y = 1/x$, not recognising the function as a shifted basic function or not knowing how the addition of the constant would change the hyperbola. The next most common mistakes were attempts to sketch the function from joining plotted points (6%) and incorrectly identifying asymptotes (6%), most commonly by sketching with the x-axis as the asymptote, even when drawing the correct asymptote at $y = 1$. The visual barrier of the solid line seemed to overwhelm the upward shift of the asymptote. Curiously, many of the functions sketched from plotted points were drawn as straight lines, indicating an overgeneralization of the straight-line formula $y = mx + c$. Partly-correct answers were given by 4% of the students who sketched the correct shape but did not indicate asymptotes.

A very small percentage (1%) of students sketched curves that broke the vertical line test for functions. This could be indicative of a misconception of what the graph of $y = 1/x$ looks like or in one case, using a slant asymptote of $y = - x$. Possibly care was not given in sketching here. Less than 1% of students sketched only one branch of the hyperbola. Only 1 student sketched the hyperbola reflected in the x-axis and then shifted up, possibly seeing $1/x$ as x^{-1} and incorrectly interpreting the role of the negative power, although correctly interpreting the vertical shift.

TABLE 5: Student responses to sketching a new function from a basic function

Number of students	94	69	13	13	9	4	2
Response	Correct	Blank	Plot and join points with lines	Incorrect asymptote	Asymptote not shown	Not a function, c-shape	One-sided graph

A possible cause of the difficulty students had with producing a graphical image of an equation is the limited demand on students to produce a sketch of a function in high school examinations. The availability of technology may provide valuable opportunities to use class time for students to explore and develop their graph sketching visualisation. The ability to move between visual (graphical) and verbal (algebraic) representations of the same concept is a key skill in problem-solving and Fourth Industrial Revolution skills and therefore worth developing.

Calculus teachers cannot assume that students have the ability to visualise or quickly sketch a shifted basic function. As a result, calculus teaching that involves discussions of minima/maxima, concavity and asymptotes should be well supported by graphs generated by the teacher or software. Limitations of graphing software (for example, not showing vertical asymptotes) may provide opportunities for students to use mathematical terms to discuss graph characteristics, which can help develop understanding (Berger, 2013). Teaching should encourage moving away from plotting and joining points and challenging students to visualise the shape of the function by recognising the associated properties, in this case of the rectangular hyperbola these are the centre, vertices, asymptotes and focal points. Since testing increases learning due to additional exposure, transfer-appropriate processing and motivation, (Yang et al., 2021), using frequent, short questions such as matching equations to graphs in teaching and assessment could develop graph sketching skills. Since developing this skill may take more time than is allowed in a fast-paced university curriculum, making use of graphing packages like Desmos or typing a function into the Google search bar may help students develop the skill of visualising graphs of functions. Initially including a computer-drawn small sketch of functions used in lectures and tutorials would help to develop the skill of linking an equation to its graph.

Conclusion

This preliminary study addressed the most problematic questions from a diagnostic test given to first-year engineering students. The findings were not unexpected. The value of this research is the breakdown of the misconceptions, which can guide learning and teaching in school and university mathematics. Reflections on students' performance on the full diagnostic test will lead to the next action research stages of planning and acting.

Acknowledgements

The diagnostic test was designed by Dr Rina du Randt and Professor Werner Blum and used for engineering students at the University of Johannesburg. Permission to use the test was granted by Dr du Randt, subject to keeping the test confidential.

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Covid-19 lockdown: Mathematics teachers' response to emergency remote teaching

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This study focused on the emergency remote teaching of mathematics in a resource-constrained context during the COVID-19 lockdown. This study was informed by the Protection Motivation Theory (PMT), which helps us understand humans' motivational responses to possible threats associated with their health and safety. Twenty-three South African secondary school mathematics teachers were purposively selected to participate in this qualitative study of exploratory nature. An open-ended questionnaire and follow-up telephonic interviews were used to gather the data. Thematic analysis was used to analyse the data. The findings revealed how teachers became lifelong learners as they had to adapt to the digital learning environment. Teachers also faced various challenges in transitioning from face-to-face classrooms to emergency remote teaching. At the time of writing the article, most learners in South Africa had started going to school regularly. Nevertheless, this study focused on the inequalities in the South African education system laid bare by the COVID-19 pandemic.

Introduction

In March 2020, various governments across the world implemented mitigation measures in response to the COVID-19 pandemic. These measures included social distancing, which led to school closures. Consequently, around 1.5 billion learners worldwide were left without education. On 27 March 2020, a national lockdown was implemented in South Africa to contain the pandemic. Under the lockdown, the South African government insisted on "rescuing the school year" and encouraged teachers to perform some form of emergency remote teaching (ERT). ERT is a temporary shift in teaching that happens when a crisis occurs. It is different from traditional online teaching and learning. ERT "involves the use of fully remote teaching solutions for instruction or education that would otherwise have been delivered face-to-face or as blended or hybrid courses and that will, hopefully, return to the original format once the crisis or emergency has abated. The primary objective in these circumstances is not to re-create a robust educational ecosystem but rather to provide temporary access to instruction and instructional support in a manner that is quick to set up and is reliably available during an emergency or crisis" (Hodges et al., 2020, Emergency Remote Teaching section, para. 1). ERT can be used as both synchronous and asynchronous communication or interaction. Asynchronous learning allows participants to choose when and how they would like to participate in the learning. Asynchronous learning can occur through digital tools such as the WhatsApp Platform or discussion boards on Learning

Management Systems such as Blackboard or Moodle. Synchronous learning happens through live audio, video and live chats with instant or real-time feedback (Hrastinsk, 2008).

ERT of mathematics was a new experience for most teachers at South African public schools since many had not performed online teaching before the COVID-19 pandemic. In this regard, this study explored how mathematics teachers at public South African schools located in a resource-constrained context responded to the call for ERT during the COVID-19 lockdown. Thus, the research question under investigation was: How did South African secondary school mathematics teachers in a resource-constrained context respond to the call for ERT during the COVID-19 lockdown?

Significance of the study

At the time of writing this article, most South African schools had started implementing teaching on a rotational basis where learners were split into groups that attended school on alternate days. A rotational basis was mandatory to maintain social distance necessary to curb the spread of the COVID-19 virus. Nevertheless, the study reported in this article is vital since ERT of mathematics in a resource-constrained context underscored issues of inequality in the South African education system. Of course, the issue of educational inequality in South Africa is not new. Nonetheless, the COVID-19 pandemic has laid bare these realities of educational inequality from a different perspective which must be dealt with urgently.

Theoretical framework

The study was informed by the Protection Motivation Theory (PMT), which Ronald Rogers developed in 1975. PMT is a psychological model that explains how individuals are motivated to act in a self-protective manner when faced with a perceived health or safety threat. It has four key elements: threat appraisal, coping appraisal, response efficacy, and self-efficacy (Rogers,1975). Response efficacy refers to the belief that a specific process can mitigate the threat, and self-efficacy is a person's ability to implement the necessary actions to mitigate a threat. Floyd et al. (2000) observe that PMT can be applied to “any threat for which there is an effective recommended response that can be carried out by the individual” p.408. In this regard, this study adopted PMT to understand how South African secondary school mathematics teachers responded to the call for ERT early during the COVID-19 lockdown. PMT was used as a lens to develop the open-ended questionnaire items and follow-up telephonic interview questions and to inform the themes generated from the data.

Methodology

A qualitative research method of exploratory nature was adopted for the study. Sixty Grade 12 mathematics teachers at various public secondary schools located in a resource-constrained context in Gauteng, South Africa, were purposefully selected and invited to participate in this study via e-mails and WhatsApp messages. This study focused on teachers in public secondary schools because they are more likely to experience difficulties with online learning than those in private schools. In addition, this study focused on Grade 12 mathematics teachers because they were the ones who adopted ERT since their learners had to write school-leaving examinations at the end of the year. A total of 32 teachers consented to participate, and eventually, 23 responded to the open-ended questionnaire. The open-ended questionnaire was pilot tested before the data collection process. Follow-up telephonic interviews were conducted with seven teachers whose responses were not clear enough on the questionnaire. The telephonic interviews were transcribed and analyzed thematically together with the teachers 'responses on the open-ended questionnaire.

Findings

This study explored the teaching of mathematics in a resource-constrained context during the COVID-19 lockdown. The findings revealed various themes discussed in relation to the four PMT key elements: threat appraisal, coping appraisal, response efficacy, and self-efficacy.

Mathematics Teachers as Learners during ERT

South African mathematics teachers had to cope with ERT by becoming learners themselves. Many teachers in resource limited contexts had to adapt to digital teaching for the first time and learned to support their learners through novel teaching methods. Some teachers even learnt how to make their own "whiteboards" by pasting white paper on the walls of their homes. These teachers recorded themselves using their smartphones while teaching from the "whiteboards" as stated by teacher Harry:

> *I had to improvise. I pasted pieces of paper on the walls of my bedroom and used them like a whiteboard. I chose my bedroom to teach quietly and take clear videos of myself without my kids disturbing me. I shared the videos with learners and parents in the groups that I had created on the WhatsApp platform.*

Despite the teachers' efforts, the findings from this study were that, although they shared the recorded videos on WhatsApp, many learners either did not have smartphones or enough data to download them. WhatsApp is a messaging platform that enables individuals to send and receive text and multimedia messages.

Some teachers indicated that they became innovative and added voice notes to their PowerPoint slides at the beginning of the COVID-19 lockdown and shared them asynchronously on the WhatsApp groups with their learners. However, this strategy was quickly abandoned after teachers discovered that most of the learners did not have enough data to download the slides as stated by teacher Abedi:

> *Initially, I came up with the idea of adding voice notes to PowerPoint slides and loaded the slides on both the parents' and learners' WhatsApp groups. However, I quickly realised that*

most parents and learners did not have sufficient data to download these slides, and I quickly abandoned the technique.

Pedagogical Practices for ERT of mathematics

After the South African government signaled the need for ERT, teachers indicated that they started to experiment with various video communication platforms such as YouTube, ZOOM and Facebook to conduct some form of teaching. Nonetheless, findings from the study indicated that teachers could neither use YouTube, Facebook Live, nor ZOOM for ERT because most learners did not have sufficient data to attend the live lessons or listen to the recorded lessons. As a result, teachers used the WhatsApp platform to conduct some form of teaching. Most of the time, South African learners are familiar with the technology. At the beginning of the lockdown, teachers stated that they created videos and audio lessons and sent them to learners through the WhatsApp platform. The platform generally has no technical issues, but learners could not download the videos or audio due to insufficient data. Consequently, teachers resorted to text messaging to provide questions, solution strategies, solutions, and clarifications.

Some teachers mentioned that they had wanted to conduct one-to-one consultations using WhatsApp video calls as was done in affluent schools, but this was not possible due to learners in the context of not affording data costs. As was revealed in the following excerpt by teacher John:

Some learners require me to conduct one-on-one explanations, but they cannot afford data for video calls. However, in private schools, teachers are doing online teaching and one-on-one video consultations.

This comment by teacher John highlights the variations in affordability, income, and access to resources in the South African society since private schools moved seamlessly to online learning.

Mathematics Teachers Learning from a Larger Mathematics Education Community

Teachers' responses on the questionnaire items and the telephonic interviews indicated that they sought guidance from around the globe on implementing ERT. Teachers were able to connect with other teachers globally and gain practical guidance on how to deliver mathematical content, as noted by teacher Michael:

I joined a Facebook group from Israel that focused on implementing technology in an online environment when teaching Euclidean geometry and Analytical geometry. The group helped me a lot since I got ideas on designing various learning activities on circle geometry that I implemented on the WhatsApp platform.

Discussion

The study reported in this article explored how South African secondary school mathematics teachers responded to ERT of mathematics in a resource-limited context during the COVID-19 lockdown. The study revealed that the challenges associated with online learning are more prevalent in schools with limited resources. Many teachers could not effectively implement ERT of mathematics due to their learners' lack of digital resources such as affordable smartphones, data and internet connectivity. However, at elite schools, learners seamlessly transitioned from face-to-face learning to online learning during the COVID-19 lockdown (Black et al., 2020). This finding demonstrated that the pandemic accentuated the historical inequalities in education in South Africa (Chirinda et al., 2021).

Learners in the study context could not easily access learning materials that were made available on the WhatsApp platform by their teachers because of high data costs. This finding displayed that the COVID-19 pandemic perpetuated, or even aggravated educational inequality in South Africa. Chirinda et al. (2021) observe that the COVID-19 pandemic laid bare the long-established inherent structural inequalities in South Africa between the rich and the poor. The pandemic revealed the invisible and ignored mechanisms of inequality and allowed us to notice all the dysfunctional areas (Czerniewicz et al., 2020).

Conclusion

ERT of mathematics during the COVID-19 lockdown highlighted issues of inequalities in the South African education system, which must be dealt with urgently. Therefore, it is time to rethink how education is accessed in South Africa. At the same time, remembering that the digital divide will become more prevalent if educational access is determined solely by access to digital resources. In South Africa, the education system has been focusing on digital transformation since the 4th industrial revolution. Therefore, it was assumed that the COVID-19 pandemic would compel many schools to convert to complete online education. However, the transition did not happen because the situation for such a form of education was unavailable in resource-constrained contexts of South Africa.

The COVID-19 pandemic served as a wake-up call for the South African policymakers regarding the need for more robust frameworks for education technology for mathematics learners in under-serviced contexts. We believe that digital resources must be freely accessible to all learners in South Africa for digital transformation in education to happen. It is important to note that the South African government made some progress in addressing the access to digital resources by partnering with network providers like MTN, Cell C and Vodacom to offer zero-rated applications and educational websites to learners. However, due to the challenges in network connectivity, some learners could not use them.

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Building a community empowered with 21st Century skills within an inclusive and innovative vision through mathematics

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In this paper, we present two initiatives Wisaarkhu and UMI attempting to build a community of people with a common vision to empower the teachers and the learners with values and critical skill through mathematics that will make them successful not only in their career in the 21st century but also as human beings within their communities.

Introduction

The world is constantly changing at an increasingly fast rate. Even though the global security and level of life have improved through the years, we still observe Black Lives Matters movements, Me Too movements, climate changes movements... all over the world. We have a long way to go to offer a more sustainable future for the generations to come. To accomplish that, we need to empower our learners with the 21st century skills that they need to succeed in life and in the workplace. But success without the necessary insight and wisdom and a clear sense of community could be adding the pool of worldwide problems. Building a better future, with engaged and inspiring human beings starts in our classrooms. Mathematics classes have a significant role in the education of our learners. We need to realize how mathematics shapes our realities and our learners. We must build a community of people that can reshape mathematics to empower our learners with critical life skills that will make them the human beings we wish to see more in the world. The two projects I will present here attempt in their own distinct ways to create communities of people that are eager to create a more inclusive and innovative way to facilitate mathematics learning with a purpose and the goal to empower our learners. They are both changing me in the way they are making me self-aware of the flaws our current education system might have and they are enabling me to engage with inspiring people all over the world with a shared vision for a better future. From these engagements I learn a lot. They both make my teaching more human and more aware on the impact I have in my student's future.

Wisaarkhu

About the initiators

The project initiated with Prof. Ingrid Rewitzky, Prof. Zurab Janelidze, and me. Prof. Ingrid Rewitzky has been involved in educational leadership within and across complex environments through her roles as Professor of Mathematics, Executive Head of the Department of Mathematical Sciences, and Vice-Dean (Learning and Teaching) of the Faculty of Science. Her personal growth and satisfaction as an academic and educational leader are integrally linked to transcending generational differences and diversity with integrity, fairness, empathy, effective listening, humility, and caring. As a leader she strives to foster a collegial environment (for colleagues and students) promoting inclusivity in which each individual feels respected, trusted, motivated, and valued, all decisions are made fairly and with integrity, and excellence, loyalty, and service are acknowledged and rewarded. She is editor-in-chief of the magazine Wisaarkhu.

Prof. Zurab Janelidze is a mathematician working in category theory - a field of abstract mathematics that studies unified conceptual principles of different areas of pure mathematics. He is a professor at Stellenbosch University and serves on the editorial boards. His hobbies include drawing and classical music improvisation. He has also founded a Foundation of Abstract Mathematics course which aims to invite the students to explore mathematics in their own time and remove as much as possible the idea of assessment. He is a key person within the project with amazing ideas to develop it.

Finally, I am a researcher and a lecturer in Mathematics at Stellenbosch University. I am Franco-Portuguese married to a South African. I previously worked at the University of Padova, in Italy, University of Bordeaux I, in France, Courant Institute, in the USA as well as UCT in South Africa. Through my international roots and experience, I had to question constantly and evolve my teaching philosophy. My love for teaching started at an early age and I see research as a form of teaching. As I build my teaching experience, I have become aware of a disconnect between what is taught and day-to-day life. I manage and organize most of the components of Wisaarkhu.

The project

So many walls have been built around math, creating anxiety and discomfort. These walls disconnect the subject from reality and from other disciplines and give the impression that it lacks humanity. I realized it was difficult to make use of whatever I was teaching in everyday life. There are too many barriers between schools and universities and as we advanced in life. The project aims to break down all those walls and give everyone the opportunity to learn mathematics with the important life skills that can be transmitted through learning mathematics. Mathematics can equip us all and give everyone a fair chance to participate in society with dignity. With this project, we aim to break the stigma around mathematics.

The project organizes regular discussions involving students, lecturers from various disciplines, learners, teachers, educators ... guided by a given theme and inspired by a 10-minute talk by a very influential speaker followed by 50 minutes open discussion. The project also produced a Wisaarkhu magazine. Each year the magazine compiles at least one volume with a certain theme identified by the team as an important topic to reach our goals and make our impact. Each volume contains 30-40 columns from experts around the world from various disciplines and backgrounds, but also gives a voice to students, learners, teachers, parents, and lecturers. Each column is reviewed by at least 3 members of the editorial team to ensure the quality and validity of each column. Our aim is to give a powerful voice to each of our contributors. Each volume aims to offer to the reader many perceptive on a given topic aimed to stimulate further reflection and provides perceptive into their opinion. Opening the mind of the reader we hope to transmit the value of mathematics and change the image and status of mathematics. Each column is curated with art. We have three volumes up to now: Competition/Collaboration; Mathematics during the pandemic; and Breaking the stigma around mathematics. We also have a team of poets and artists who combine art and mathematics on our art page. Many beautiful poems are online already. The aim is for people to see that mathematics, art, and creativity are not mutually exclusive but are complementary and interdependent.

Wisaarkhu aims to provide a safe space to think about how to make contemporary mathematics more accessible to a larger community of new generation of students and researchers, and to inspire more interest in it, investigating the needs of the different parties involved. It is a challenging objective, but with many of us collaborating with different perceptive and expertise it is the start of the answer. Combining our strength with psychological lenses, we hope to reveal the human side of mathematics, defragmenting knowledge leading to more informed open minds capable of making more conscious decisions more adapted to our uniqueness and authenticity. The project is also continuously evolving as our community evolves. Our contributors are from different disciplines and backgrounds and contribute in many forms that are all equally valued. Wisaarkhu is a place where you can think about mathematics without walls, and you can connect mathematics with everything and everyone. Wisaarkhu strives to encourage people to use mathematics to attain their goals. As a lecturer, the thing I have found sad is that many students, after graduating, do not engage with mathematics any further. The project aims to change this. Our relationship with mathematics has a significant impact on how we see ourselves and society. As a lecturer, I have seen how my students have really battled with mathematics and once they persevered how they have thrived. They learned life skills such as focusing and empathy, critical thinking as well as mindfulness. In a society that is empowered with these skills and perspectives, they will learn to make sustainable choices for themselves as well as for the planet.

There are many questions that remain unanswered. Such as how do we expand our reach to disadvantaged communities and make a tangible difference in their lives. We wish our social impact to be multifaceted and work toward empowering disadvantaged communities surrounding us through education as well as provide a simple basic need to this community. It always makes me sleepless to know how someone can guide some to be their best without breaking their authentic self.

UBUNTU Mathematics Institute

About the initiators

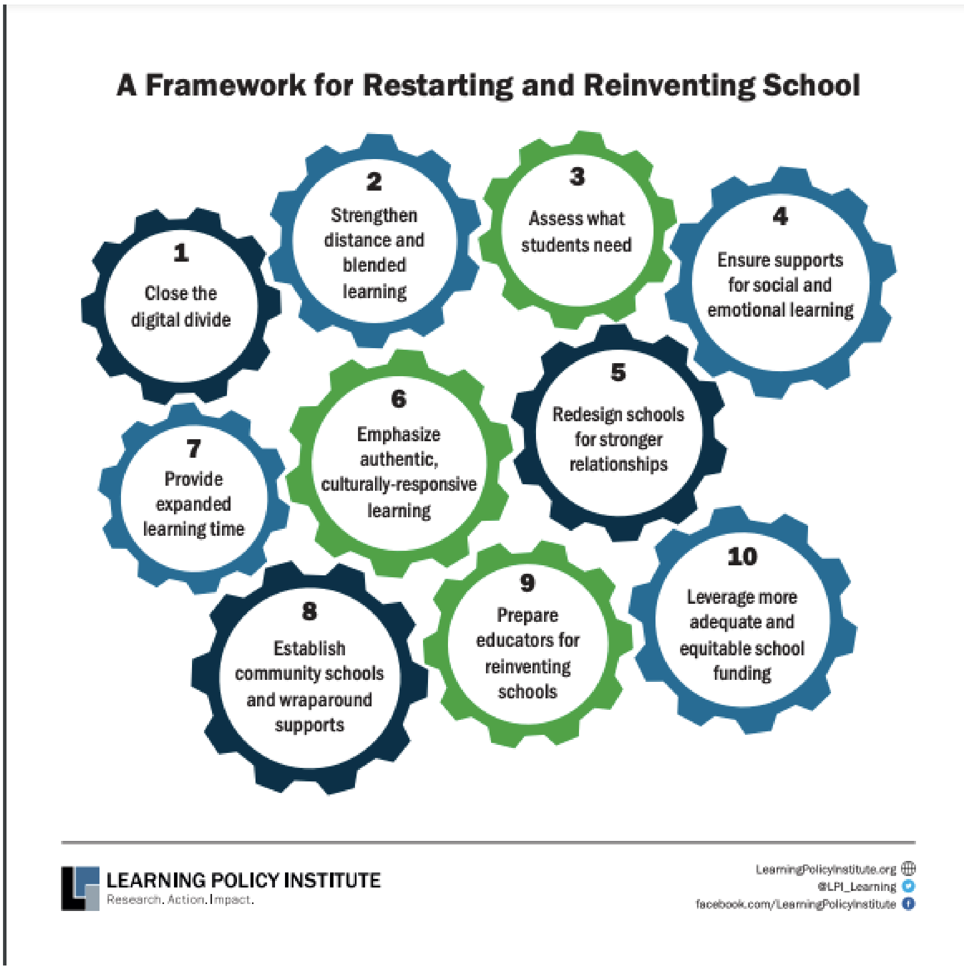
At the beginning of 2021, Dr. John David Volmink and John Gilmour initiated the UBUNTU Mathematics Institute. Both are highly active changemakers in the world of mathematics education. John Volmink has recently obtained an honorary doctorate from Stellenbosch University who stated: "Through his exceptional leadership skills, Prof John David Volmink has made an outstanding contribution to education, in particular, to curriculum reform, the advancement of mathematics education and the quality assurance of schooling in post-apartheid South Africa. Every post-apartheid Minister of Education requested him to play a leading role in the transformation of education. In addition to serving as chairperson of Umalusi, the Council for Quality Assurance in General and Further Education and Training, and various Ministerial Task Teams, Volmink has been instrumental in the establishment of NGOs that help with development and transformation in different parts of the world."

John Gilmour is an educational social entrepreneur and activist who has piloted and scaled a programme to cultivate the holistic development of marginalized children living in townships in South Africa. Incensed by the inequality in the education system and its overemphasis on cognition, he made it his mission to flip this pedagogy to integrate much neglected social and emotional learning (SEL) as part of the school curriculum, which focuses on self-liberation, self-empowerment, and the development of personal resilience. He is executive director of LEAP Science and Maths Schools and the Global teacher institute. He cofounded Bridge and The South African Extraordinary Schools Coalition but also serves in different boards. Together, they joined their passion and strength and with a founding team of interested practitioners developed a charter for the Ubuntu Mathematics Institute that explains its underlying values and will guide the members through their shared journey in reshaping mathematics education.

Motivation and context

The work of Jo Boaler plays a significant role within the values of the emerging Institute: for instance, her idea that the most damaging idea in mathematics is to think that to understand it you must have a special kind of brain; also her insistence on encouraging the “struggle” to learn mathematics; and her drive to encourage the discovery of mathematics by the students themselves. Self-belief, creativity, and valuing mistakes are key in her vision of teaching mathematics. She values learning versus performing, depth versus speed as well as questioning and discussions. She also argues the importance of visualization and making connections to strengthen our brain.

The framework for restarting and reinventing school put in place by the Learning Policy Institute holds a prominent place in the vision of the Institute.



The Adoption of Ubuntu in the Teaching and Learning of Mathematics and Science is one of the innovative ideas of the Institute (see also [1]).

The Institute's philosophy anchors itself into the context in which it lives: South Africa's declining mathematics performance [2]; decolonization of mathematics [3]; but also, recent research in education such as [4].

The Charter

The purpose of the Institute is to work together to inspire, challenge, and empower all teachers and education leaders. All teachers should embrace mathematics as a non-negotiable part of the learning for all children of Africa. The Institute aims to transform instructional practice to focus on making it child-centered, competency-developing, and character-building. In doing so, it will make use and apply the latest relevant research, technology, and tools to make mathematics learning accessible, valuable, meaningful, and human.

The member of the Ubuntu Maths Institute share a vision of quality education for all children in Africa as well as equitable access to teaching and learning that liberates and facilitates the emergence of fully conscious, competent, character-defined, activated community and global citizens and gender equitable expectations and learning pathways. It aims for all children and schools embracing and living the values of UBUNTU. One of the main purposes of the Institute is for all children learning to love the processes of exploring and using mathematics. An important belief of the Institute is that all students can learn mathematics to high levels. To achieve that, changing teachers' fixed mindsets is the core lever to changing the quality of teaching and learning. Mindset shifts must be reflected in the commitment of teachers and whole schools' commitment for optimal impact. Mathematics teaching and learning in Africa will thrive within the framework of the shared humanly interdependent values of UBUNTU.

To reach its goal, the Institute will need to develop and share a full understanding of 'growth mindset, working hard to challenge and change 'fixed mindset' assumptions and beliefs relating to the teaching and learning of mathematics. The Institute supports, develop, and align facilitated learning processes that are underpinned by the values and practices of Ubuntu and high expectations of students. It needs to contextualize and apply perspectives of learning in African reality. It will find ways to implement experiential learning through integrated practice and project-based learning. The power of peer learning, peer support and shared accountability will be a key part of the process as well as prioritizing the development of metacognition. The Institute aims to develop ways to increase students' love of mathematics. Excellence will be at the center of the Institute that will promote achievement of mastery of the mathematical literacies, development of the necessary competencies and effective use of technology as an enabler.

It is no surprise that the Institute's members realize the embedded constraints. On the one hand, modern research emerging as new science on the brain and learning, giving important insights into mathematics learning, is not widely available nor actively shared. On the other hand, the fear and intimidation of mathematics in Africa, accompanied by the fear of failure is reflected directly by historically embedded colonial patterns of protecting privilege. This is now continued in the patterns of economic and social inequality. Moreover, current mathematics learning and teaching methods are not adjusted to producing citizens for the 21st century and beyond. In most school contexts Mathematics is taught in a silo of learning often completely disconnected from any application or contextualizing value. Mathematics teaching methodologies remain stuck in transmission mode with teacher-centered focus and learning and assessing under time pressure.

But the Institute has a plan of action. It starts with creating an open Institute community identifying/creating/amplifying/sharing/supporting research-based innovative teaching methods, mathematics tasks, videos, and ideas intended to reduce mathematics' failure, avoidance, fear embedded in the learning poverty of South Africa and beyond - humanizing mathematics teaching and learning; inspiring teachers and empowering all students to success.

With the adoption and implementation of the Ubuntu Maths Institute mindset and practice shifts, we expect the students to develop the following 21st century skills:
1. Literacies - numeracy, ICT (Information and Communication Technology), scientific, financial, cultural, and civic…,

2. Competencies - critical thinking, creative thinking, communicating, and collaborating… and

3. Character - curiosity, initiative, persistence/grit, adaptability, leadership, social and cultural awareness. Those skills being the 21st century skills identified by the World Economic Forum in their new vision for education in 2015.

Conclusion

There is still a long way to go to make mathematics accessible for all in an equitable way as well as transforming the experience of students with mathematics into something empowering and valuable in their lives. The magnitude of the task is too big for a single person and creating communities/institutes such as Wisaarkhu or UMI with this shared vision, that inspires innovative and constructive ways to build the future is the way forward to create an unstoppable momentum that achieves positive change for the future.

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Reflecting or not reflecting: Secondary Mathematics teachers' perspectives

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Reflecting on practice is a paradigm that dominates teacher education around the world, and most professional teacher intervention programs include it to improve teachers' practice. This article provides an analysis of secondary mathematics teachers' perspectives of reflecting on their practice within an intervention programme in the province of KwaZulu Natal. Data presented here was generated from thirteen secondary mathematics teachers in two districts in KwaZulu-Natal who participated in the JIKA IMFUNDO project. Data were collected using secondary data, interviews, and document analysis. Findings revealed that the mathematics teachers see little or no value in reflecting on their practices. They consider reflections as unnecessary administration tasks that do not contribute to their professional growth and learner performance. Based on the findings, researchers recommend that teachers be guided to design reflection tools that are meaningful to them and their context.

Keywords: Reflections; secondary mathematics teachers; reflective practice

Introduction

JIKA IMFUNDO is an intervention programme launched by the Programme for Improving Learning Outcomes (PILO) in partnership with the KwaZulu-Natal Department of Education (KZN DoE). The central principle that informs JIKA IMFUNDO is that if curriculum coverage improves, then the learning outcomes will be improved. Thus, the overall aim of JIKA IMFUNDO is to provide all education stakeholders with appropriate tools to enable them to initiate professional and evidence-based conversations about curriculum coverage. One of the JIKA IMFUNDO tools is the curriculum planner and tracker (CPT) for teachers to track the curriculum. In the process of improving teaching and learning, tracking the curriculum, teachers are encouraged to develop the skills of being a reflective practitioner of one's teaching by completing weekly reflections and end-of-term reflections. The purpose of reflections in CPT is for teachers to have a professional conversation with themselves and HOD on how to improve learning and teaching.

Research indicates that one of the significant elements of improving teaching and learning in education is for teachers to be reflective practitioners (e.g Dewey, 1993; Farrell & Ives, 2015). Cimer, Cimer, and Vekli (2013) state that reflection upon teaching practice will more likely enhance teacher professional growth and discipline knowledge and probably lead teachers to understand the pragmatics of classroom instruction. Although there is a plethora of literature (e.g., Wallace, 1991; Dewey, 1993; Reed et al., 2002) that postulate the importance of teachers becoming reflective practitioners, limited research focus on the extent to teachers see the value of reflection in their classroom practices and professional growth in South Africa. This article, therefore, explores mathematics teachers' perspectives about reflections within the JIKA IMFUNDO intervention programme. With this in mind, we set to answer the following research questions: *How do mathematics teachers who participated in the JIKA IMFUNDO programme perceive reflections for practice?*

Related literature

The improvement and development of teacher practices is considered an ongoing practice not only in South Africa but globally. Burke (2002) posits that how the teachers interpret what is happening in their practice and how they react to the process determine how they transform and implement the change. This means that teachers need to value any initiative set to improve their practice for them to adopt and implement it.

Reflection is a vital skill that is considered a prerequisite for teacher development and has been defined in various ways in the literature, most commonly from a cognitive perspective to including experiences, values, and emotions. For example, Leijen, Lam, Wildschut, and Simons (2009) consider reflection a cognitive process or activity because it allows the creation of knowledge about one's cognition, and regulation of that cognition. The purpose of reflection in a professional context is to firstly, bring about change to improve practice and to develop self-knowledge and understanding (Sarivan, 2011; Sellars, 2012; Akbari, 2007). Sarivan (2011) considers reflection to be useful for one's professional development and practices and emphasise the importance of documenting it for reflective practices. Through reflection, teachers can attain self-knowledge and improve their practice by utilizing the notion of intrapersonal intelligence as purported by Gardner (1993). On the same vain Day (2015) posits

that reflection is necessary for all teachers to maintain their effectiveness by writing and through engaging in reflective practices daily they are more likely to understand the effect of their motivations, prejudices, and influences in the lives of the learners they teach. Reflecting in action as termed by Schon 1983 (cited in Bansilal and Rosenberg; 2011) has the potential to reduce the dangers of forgetting what actually happened during actions and thus improve the teaching in the lesson (Cimer, Cimer, and Vekli, 2013).

While reflections have been identified as one of the factors to improve teaching and learning and the importance of teachers to be reflective practitioners have been argued for by many scholars (e.g Day, 2015; Cimer et al, 2013, Bansilal and Rosenberg, 2011), Cimer et al (2013) cautioned against factors that might hinder teacher reflections. Authors argued that teachers' emotions and working conditions are the contributing factors to teachers either reflecting or not reflecting. It is known that a teaching job does not only entails being in class and transmitting knowledge it is an emotional exercise thus teachers' emotions need to be catered for in the quest to enhance teacher practices. Therefore, Day (2015) cautioned that ignoring the place of emotion in reflection, in, on, and about teaching and learning is to fail to appreciate its potential for positively or negatively affecting the quality of the classroom experience for both teachers and learners. While there is a plethora of literature indicating that reflections can bring about change to support teacher development, Zeichner and Liu (2010) argue that there is little evidence that reflection supports positive teacher development. They further argue the failure of reflections to promote teacher development is due to the emphasis on teachers' reflecting inwardly on their teaching and learners and neglecting consideration of the social conditions of schooling that also influence the teacher's work within the classroom.

Methodology

This article adopts a qualitative research design because it provides an overall lens for the study in question and becomes an advocacy perspective that shapes the types of questions asked and informs how data are collected and analysed (Creswell & Poth, 2016). While there were one hundred and seventy-six schools that participated in four JIKA IMFUNDO surveys, data presented here was generated from fourteen schools. The schools were selected based on their participation in at least three JIKA IMFUNDO surveys which utilised questionnaires to collect data. Furthermore, document analysis for the utilisation of the curriculum planner and tracker was conducting in the fourteen schools selected. In each school one teacher was interviewed, however in one school a teacher did not consent to be interviewed thus data presented here was generated from thirteen secondary mathematics teachers. Since JIKA IMFUNDO was introduced in grade 8 and 9 in 2015 and in grade 10 in 2016, the criteria for selecting teachers was that teachers should either be teaching grade 8 or grade 9 or grade 10.

Data analysis

The analysis of the secondary data involving counting the number of schools that indicated that teachers are reflecting or not reflecting. After the analysis three categories emerged, that is reflections done, reflections done minimally, and reflections not done. The details of the categorise are elaborated on in the finding’s sections. To verify what transpired in the secondary data curriculum planner was analysed. The same categories established when analysing secondary data were used. As a follow, up semi-structured interviews were conducted with teachers, however, only 13 secondary mathematics teachers who gave consent to be interviewed. Data from the interviews were analysed inductively through coding and generating themes. Data was first transcribed and manually coded. Codes with the same meaning were collapsed into one code to generate themes. The final step involved synthesising the themes, which were arranged to align with research questions. To interpret how teachers in the programme perceive reflections two themes emerged that is reflecting is time-consuming and a duplicative exercise that wastes their time. These themes are elaborated on in the finding's sections

Findings and Discussion

The perennial question guiding this study was to explore secondary mathematics teachers' perspectives about reflecting on their practice. Figure 1 below presents the section teachers needed to complete in the curriculum planner and tracker about reflection.

Reflection		
Think about and make a note of: What went well? What did not go well? What did the learners find difficult or easy to understand or do? What will you do to support or extend learners? Did you complete all the work set for the week? If not, how will you get back on track?	What will you change next time? Why?	
	HOD:	**Date:**

Figure1: Extract from curriculum planner and tracker (source Department of Education, 2015

In the secondary data, data drawn from 2015 survey, showed that mathematics teachers in the selected schools consider themselves to be reflective practitioners because in the surveys the either tick yes or partial when asked about reflection practices. Data drawn from 2016 Surveys revealed that secondary mathematics teachers also considered themselves to be reflective practitioners. This was based on the colour codes use when responding to questions about reflection practices. If teachers do complete reflection section, schools had use green colour, if teachers do it but not regularly the schools should rate itself as amber, if mathematics teachers are not reflection the school should rate itself red. The analysis showed the schools either rated themselves green or amber, thus showing that mathematics teachers are reflecting on their practice.

Contradictory to the findings from the secondary data, the analysis of the curriculum tracker revealed that only four secondary mathematics teachers in these fourteen schools are completing the reflection section. Out of the four schools whose teachers completed the reflection section, only two showed evidence of doing it weekly as specified in the curriculum tracker. The evidence presented by secondary mathematics teachers from the other two schools showed that the reflection is not done on regular basis. Although in the survey it seemed all mathematics teachers in the fourteen schools are reflecting, the evidence obtained from analysing the curriculum tracker proved otherwise since only two are reflecting routinely. The findings suggest that reflecting in action is not being done by teachers in the programme. In the quest to understand the discrepancies between what transpired from the secondary data and document analysis we set to understand secondary mathematics teachers' perspectives about reflections.

Reflecting is time-consuming.

While teachers acknowledge the importance of reflections, the findings revealed that secondary mathematics teachers do not reflect on the successes and challenges of their lesson because they find it to be time-consuming. One secondary mathematics teacher felt that the department of education does not understand the severe conditions under which they teach, which is why they were expected to do so much paperwork: *"I think the department has lost track of what is happening at the grass-root level*." He further indicated that he felt that reflections do not allow him to raise honest opinions: "*What you did and what you enjoy and what you will do better next time is nonsense. We never enjoy anything, we are struggling to get concepts across to learners"*.

The comments by the above secondary mathematics teacher resonate with the argument by Cimer et al (2013) that teachers' emotions and working conditions need to be considered as they have the potential to hinder teachers' willingness to reflect. As the above secondary mathematics teacher articulated that he does not enjoy anything and therefore does not see the need to reflect.

On the issue of too much workload and time-consuming that comes with reflections, Sarivan (2011) argues that success in teaching is not just a matter of luck but results from thorough planning and preparation, knowing your learners, and reflection on an evaluation of, practice. He further argues that reflection is a process and an activity that teachers undertake primarily for themselves and should not be regarded as hard work or wasting time. Therefore, teachers must see the value of reflections for it to be meaningful for their practices. However, this seems not to be the case with some of these secondary mathematics teachers.

Reflecting is a duplicative exercise and a waste of time.

According to Dewey (1993), the attitude required for reflection is whole-heartedness, which indicates the enthusiasm of an individual for his or her subject matter. However, the findings of this study showed that secondary mathematics teachers find reflection to be a duplicative exercise. Secondary mathematics teachers echoed these sentiments

"*Department wants a lot of documents with reflections, in lesson plans, we must reflect, in curriculum tracker, we must reflect; why should I produce evidence that I am reflecting if it is for my own growth*? One teacher further said this

"*Usually I do not write reflections. I know it's important to write down my reflections but there is no time to do it".*

When probed further about the importance of being a reflective practitioner, the one secondary mathematics teacher further said, "not *writing reflections does not mean I do not reflect it just means I do not have time to write it down and has nothing to do with my professionalism or my job.*"

Another secondary mathematics teacher commented that

"*writing reflections does not mean I engage with classroom issues doing it because it is expected"*

Secondary mathematics teachers' perspectives contradict the argument by Sarivan (2011) of the importance of being a reflective practitioner. Sarivan stresses that reflection needs to be documented since reflective practice is a professional requirement that must provide evidence and argue that documenting reflections help teachers identify development points for action planning. In addition, the findings revealed that routine reflections as purported by Day (2015) are not being valued by the teachers in the programme as they seem not to see a need to document their reflections and to reflect routinely.

Conclusion and Recommendations

To answer our researcher questions, the findings reveal that while on paper teachers do say they are reflecting there is little evidence to support their claim. The findings reveal that teachers in the programme see little or no value in reflecting on their practices. Although literature emphasises the importance of reflecting in one's practice, findings of this study showed that some secondary mathematics teachers are of a different view. Although it is possible to reflect without writing down, written reflections enhance meaningful professional conversations between colleagues. The findings further reveal that secondary mathematics teachers perceive reflecting as not contributing to their professional growth or enhancing teaching rather they view it as a stressful exercise that adds burdens to their already difficult working conditions. As mentioned by Burke (2002) teachers need to see value for them to implement change, therefore it is critical to encourage teachers to design reflecting tools that are meaningful to them and their context. Based on the findings of this study we recommend that a larger study need to be done to interrogate why mathematics teachers do not find value in doing lesson reflections.

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Assessment as learning vs assessment for learning

Benadette Aineamani

Pearson South Africa

In this workshop, we explore the difference between assessment as learning and assessment for learning. I will draw on worksheets from Pearson's Navigation packs to categorise tasks as suitable for assessment for learning or assessment as learning, drawing on the literature and understanding of the two types of assessments. The focus is on Grade 7 and Grade 10 worksheets; however, the workshop will benefit all mathematics teachers of all phases. Hailikari, Nevgi and Lindblom-Ylänne (2007) argue that any assessment should bring value to both parties, the assessor and the learner. In other words, the assessment should be informative and have enough detail to guide and support instruction (Dochy, 1996). Hailikari et al. (2007) also argue that a good assessment should distinguish between different types of knowledge, the knowledge of 'what', 'how' and 'why'. Valencia, Stallman, Commeyras, Pearson, and Hartman (1991) argue that various forms of assessments may be used to capture a holistic view of learners' prior knowledge and understanding in the process of teaching and learning. The types of activities that are selected for a given type of assessment are important, as we will explore and discuss in this workshop.

Target audience

Applicable to all grades

Duration

1 hour

Maximum number of participants

You may limit the number of participants in your workshop. Workshop presenters should attempt to cater for at least 30 participants.

Motivation for the workshop: Why is the workshop important? How will it help participants?

The role of assessment in the teaching and learning of mathematics cannot be overemphasised. In this workshop, participants will gain insights into the two types of assessment: Assessment for learning Vs Assessment as learning. The knowledge and insights from the workshop will benefit the teachers in their lesson planning.

Description of content of workshop

In this workshop, we explore the difference between assessment as learning and assessment for learning. Hailikari, Nevgi and Lindblom-Ylänne (2007) argue that any assessment should bring value to both parties, the assessor and the learner. In other words, the assessment should be informative and have enough detail to guide and support instruction (Dochy, 1996). Hailikari et al. (2007) also argue that a good assessment should distinguish between different types of knowledge, the knowledge of ‘what’, ‘how’ and ‘why’. Valencia, Stallman, Commeyras, Pearson, and Hartman (1991) argue that various forms of assessments may be used to capture a holistic view of learners’ prior knowledge and understanding in the process of teaching and learning.

What will be done in the workshop? How will the time slot be broken up?

Focus	Content	Time allocation
Definition	-What is assessment for learning? -What is assessment as learning?	3 mins
Characteristics	Characteristics of the two types of assessments	7 mins
Activity: Analyse activities	Analyse and categorise the activities	20 mins
Feedback	Feedback from the group analysis activity	15 mins
Summary	Summary of the activities	5 mins
Conclusion	The role of the two assessments in teaching and learning	10 mins

The activities and worksheets to be used in the workshop (maximum 8 pages)

Workshops need to be hands-on sessions where participants are actively involved in doing the activities that you provide. Usually these activities will be done in groups, consisting of 3–5 participants. There should also be ample time for discussions (approximately 25% of your time is suggested). If you have used ideas from other sources, it is essential that you acknowledge these sources.

AMESA
Workshop_Assessmer

AMESA
Workshop_Assessmer

Problem Solving

Gloria Mthethwa

Pearson South Africa

The session covers skills that are needed to solve all different Mathematics problems related to diverse topics. The emphasis is put on the practical strategies to improve critical thinking in deciding on which basic operations to use, how to use and understand Mathematical language (word problems), progression adhered to within the grades in the Intermediate Phase, four steps to be followed when solving mathematical problems. The session is interactive, and we look at using concrete objects in activities to solve mathematical problems (brainteasers) and introduce games to solve Mathematical problems.

Target audience
Intermediate Phase

Duration
1 hour

Maximum number of participants
You may limit the number of participants in your workshop. Workshop presenters should attempt to cater for at least 30 participants.
30

Motivation for the workshop: Why is the workshop important? How will it help participants?

The workshop explores mathematics within a problem context and encourages participants to think mathematically to find solutions to a problem.

The workshop will provide participants with broader understanding of mathematics problem solving and how to teach it using different strategies, ability to make choices, model, and investigate problem situations.

Description of content of workshop

The session covers skills that are needed to solve all different Mathematics problems related to diverse topics. The emphasis is put on the practical strategies to improve critical thinking in deciding on which basic operations to use, how to use and understand Mathematical language (word problems), progression adhered to within the grades in the Intermediate Phase, four steps to be followed when solving mathematical problems. The session is interactive, and we look at using concrete objects in activities to solve mathematical problems (brainteasers) and introduce games to solve Mathematical problems.

What will be done in the workshop? How will the time slot be broken up?

It is a workshop that allows participants to interact and participate in activities presented.

- Session introduction: 5 min
- Aims and objectives of workshop: 5 min
- Brain teaser: 5 min
- Problem solving process: 20 min
- Strategies: 15 min
- Examples of strategies: 30 min
- Activities: 30 min
- Final remarks: 10 minutes

The activities and worksheets to be used in the workshop (maximum 8 pages)

Workshops need to be hands-on sessions where participants are actively involved in doing the activities that you provide. Usually these activities will be done in groups, consisting of 3–5 participants. There should also be ample time for discussions (approximately 25% of your time is suggested). If you have used ideas from other sources, it is essential that you acknowledge these sources.

Participants will respond to activities given out on presentation slides, additionally there will be a link to questionnaires which will be posted on the chat for each participant to answer and submit immediately online to the presenter. Extracts and activities are taken from Smart-Kids books. Smart-Kids is a series of CAPS-aligned workbooks that help learners develop key mathematical skills. Below are a few examples to illustrate the type of activities we will be covering in the workshop.

Problem Solving Strategy: Act it out

Solution

Let five learners play the parts by standing in a line and organising themselves in the following way:

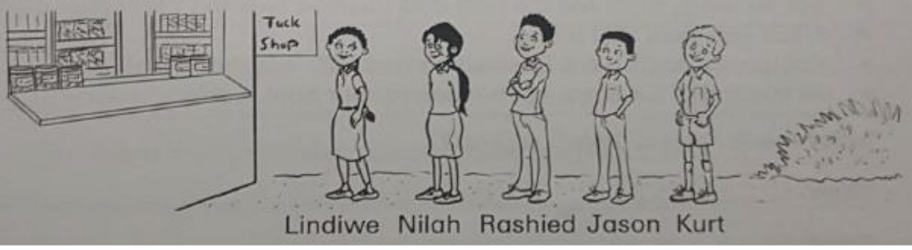
- **Step 1**: Kurt must stand behind Jason.

- **Step 2:** Rashied must stand between Nilah and Jason.

  Nilah must be in front of Jason because we know that Kurt is **right behind** Jason. So Nilah is now first, Rashied is second, Jason is third and Kurt is fourth.

- **Step 3:** Nilah must stand behind Lindiwe which means that Lindiwe must stand in front of Nilah. This means that Lindiwe is first, Nilah is second, Rashied is third, Jason is fourth and Kurt is fifth.

  Answer: Kurt is at the back of the line.



Example

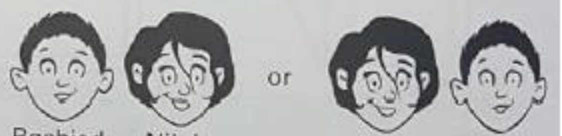
Nilah, Kurt, Lindiwe, Rashied and Lebo are sitting next to each other.

Nilah Lindiwe Kurt

Who is sitting next to whom?

Solution

- **Step 1:** Nilah is sitting next to Rashied. We do not know if Nilah is sitting to the right or left of Rashied.



Rashied Nilah Nilah Rashied

- **Step 2:** Lindiwe is to the right of Rashied. Now we know that Nilah is sitting to the left of Rashied (because Lindiwe is sitting to his right).

Nilah Rashied Lindiwe

- **Step 3:** Kurt is between Lebo and Nilah. This means that Kurt is to the left of Nilah, and Lebo is to the left of Kurt.

So the 5 friends are sitting in this order: Lebo, Kurt, Nilah, Rashied, Lindiwe.

Lebo Kurt Nilah Rashied Lindiwe

Act it out grade 5

Rashied has a 2 litre bottle of fizzy cola. He pours half of the Fizzy cola out into a large jug, and then gives the bottle to Lindiwe. Lindiwe then pours half of what's left in the bottle into a smaller jug. Then she hands the fizzy cola bottle to Kurt. Kurt pours half of what is left in the bottle into a glass.

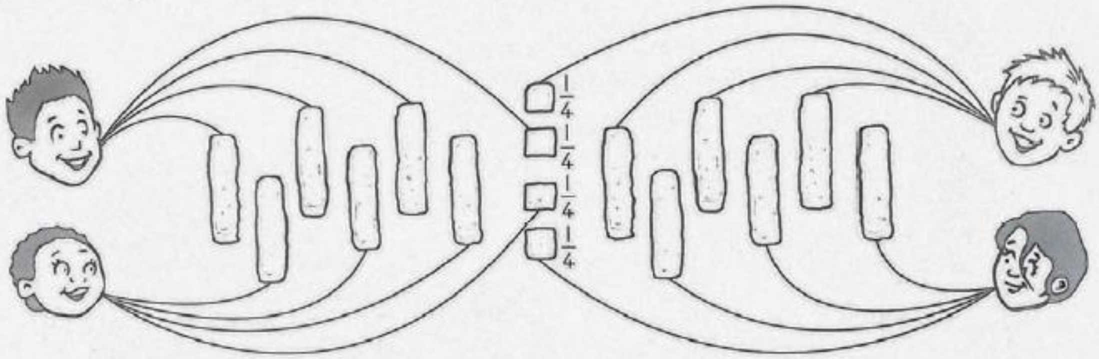
1. How much Fizzy Cola is left in the bottle?

2. What is the fraction of 2 litres ?

Problem Solving Strategy: Draw a picture/sketch

Example

I have 13 fruit bars that I would like to share equally among four children. How many fruit bars does each child get?



Solution

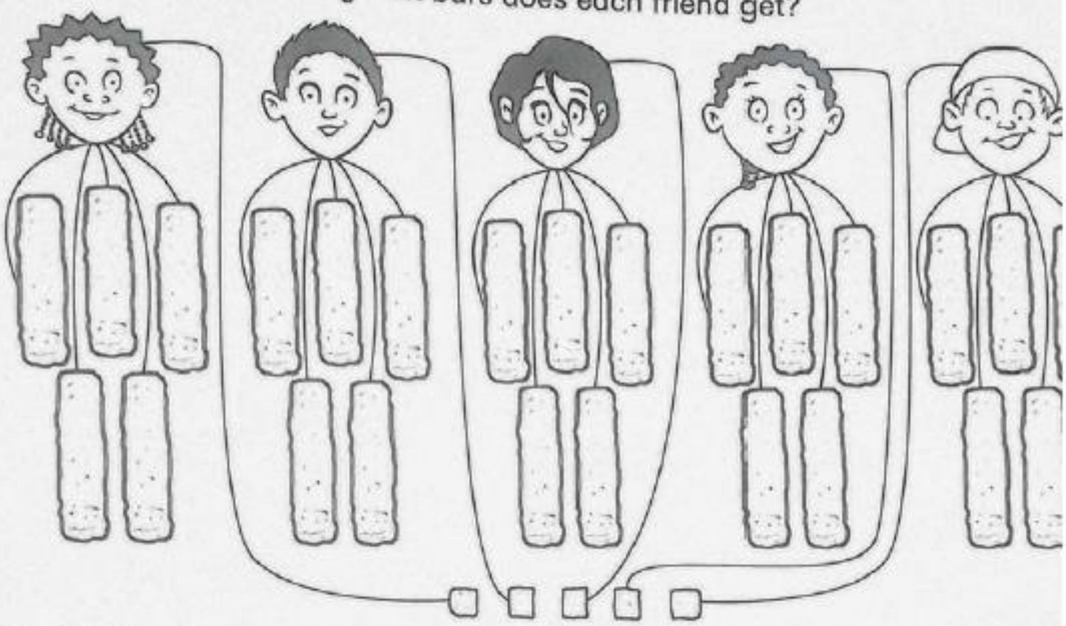
Each child gets three fruit bars and one quarter of a fruit bar.

Therefore each child gets $3\frac{1}{4}$ fruit bars.

Check: 4×3 fruit bars = 12 fruit bars

12 fruit bars + four quarters of a fruit bar = 13 fruit bars ✓

Kurt's mom has 26 fruit bars that she wants to share out equally among five friends. How many fruit bars does each friend get?



Solution

Each child gets five fruit bars plus one-fifth of a fruit bar.

So, each child gets $5\frac{1}{5}$ fruit bars.

Check: 5×5 fruit bars = 25 fruit bars
25 fruit bars + $\frac{5}{5}$ of a fruit bar
= 25 fruit bars plus 1 whole fruit bar
= 26 fruit bars

Grade 6 activity

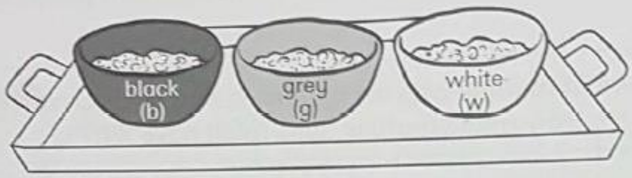
- In the grade 6 classroom, the desks are organised in equal rows. Jody sits at the desk that is 4th from the back. There are 4 desks on the right but only one to the left of Jody's desk. How many desks are in the classroom?

	1				
	2				
	3				
1	Jody	1	2	3	4
	2				
	1				

Problem Solving Strategy: Make a list

Example

A black bowl, a grey bowl and a white bowl are arranged in a row on a tray.



a. List the other ways you could arrange the bowls on the tray.

b. How many different arrangements are there in total?

Solution

a. The first arrangement is b, g, w.
If we keep the black bowl on the left, the second arrangement is b, w, g.
If we put the grey bowl on the left, we get g, w, b and g, b, w.
If we put the white bowl on the left, we get w, b, g and w, g, b.

b. There are six different ways to arrange the bowls on the tray.

Grade 6 - Make a table and look for a pattern

A choir competition is being held in the school hall. The first member of the audience arrived on her own. Then a group of three friends arrived together. Next, five people arrived in a group. After that each time a group of people arrives, there are two more than in the previous group. Use a table to work out how many people will arrive in the 20th group

Solution

Draw a table consisting of 2 rows and 21 columns. Make the heading of the first row 'group number'. Make the headings of the second row 'number in the group'

Group no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
No in the group	1	3	5																	

Group no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
No in the group	1	3	5	7	9	11	13	1	17	19	21	23		27	29	31	33	35	37	39

Problem Solving Strategy: Eliminate possibilities

Example

Kurt asked his Uncle Scott how old he was. His uncle gave him three clues so that Kurt could guess his age.

How old is Kurt's Uncle Scott?

Clue 1: I am between 19 and 30.

Clue 2: I am a multiple of three.

Clue 3: I am an even number.

Solution

Step 1: Start with Clue 1 and write down all the numbers between 19 and 30: 20, 21, 22, 23, 24, 25, 26, 27, 28, 29.

Step 2: Go next to Clue 2 and then circle the multiples of three between 19 and 30: 20, 21, 22, 23, 24, 25, 26, 27, 28, 29.

Step 3: Go to Clue 3 and underline all of the even numbers: <u>20</u>, 21, <u>22</u>, 23, <u>24</u>, 25, <u>26</u>, 27, <u>28</u>, 29. The only number that is circled and underlined is 24.

Example

Mandla, Kurt, Jaco, Lindiwe, Nilah and Rashied went bike riding. They parked their bikes in a row at the park. Work out the order in which they parked their bikes using the following clues:

Clue 1: Kurt is parked to the right of Jaco.

Clue 2: Mandla is parked on the end but his bike is not furthest to the left.

Clue 3: Jaco is not parked on the end.

Clue 4: Lindiwe's bike is parked next to Mandla's.

Clue 5: Rashied is parked next to Lindiwe.

Solution

1. Draw six lines to represent the names of the six bike riders.

   ______ ______ ______ ______ ______ ______

2. Read through the clues and fill in the names as you go through them.

   Step 1: Clue 2 states that Mandla is parked on the end. It is not furthest left so it must be furthest right.

   ______ ______ ______ ______ ______ Mandla

   Step 2: Clue 4 states that Lindiwe is parked next to Mandla.

   ______ ______ ______ ______ Lindiwe Mandla

   Step 3: Clue 5 says that Rashied is parked next to Lindiwe

   ______ ______ ______ Rashied Lindiwe Mandla

   Step 4: From Clues 1 and 3 we know that Jaco is not parked at the end and that Kurt is parked to the right of Jaco. This means that Jaco is in Place 2 and Kurt is in Place 3. So, Nilah must be in Place 1.

   Nilah Jaco Kurt Rashied Lindiwe Mandla

Workshop: Geometry of straight lines

Kopano Moroko

Pearson South Africa

The session has taken into consideration the revised ATPs. In the session we take a closer look at techniques teachers can use to teach the topics of relationship between angles formed by perpendicular, intersecting, and parallel lines, and solving problems using the relationship between angles formed by perpendicular, intersecting, and parallel lines.

No statement is to be given without a reason, to prompt active thinking and reasoning and ascertain understanding, whilst establishing the proper implementation and application of rules, in this way dispelling any misconceptions and confusion. Participants are to be reminded, of the importance of this topic in Mathematics, as it serves as the base learning for Geometry and Euclidian Geometry in the higher grades.

Participants get to reflect, discuss and share views, based on the workshop content, their own teaching, viewpoints gained from others, discoveries made from the session, other teaching methods in addition to their own and decide on a way forward for their own classes. The session is highly interactive, as it should be, in every class. Participants are encouraged to participate in all activities.

Target audience
Senior Phase

Duration
1 hours

Maximum number of participants
You may limit the number of participants in your workshop. Workshop presenters should attempt to cater for at least 30 participants.
30

Motivation for the workshop: Why is the workshop important? How will it help participants?

The workshop will assist participants to:

- improve their pedagogical content knowledge and subject knowledge.
- build educator competencies in facilitating Mathematics teaching and learning.
- improve confidence in teaching Geometry of Straight Lines for Grade 9.
- provide exposure to the different ways of teaching Geometry of Straight Lines for Grade 9.

Description of content of workshop

The session has taken into consideration the revised ATPs. In the session we take a closer look at techniques teachers can use to teach the topics of relationship between angles formed by perpendicular, intersecting, and parallel lines, and solving problems using the relationship between angles formed by perpendicular, intersecting, and parallel lines. The session is interactive and teachers are encouraged to participate in the activities.

What will be done in the workshop? How will the time slot be broken up?

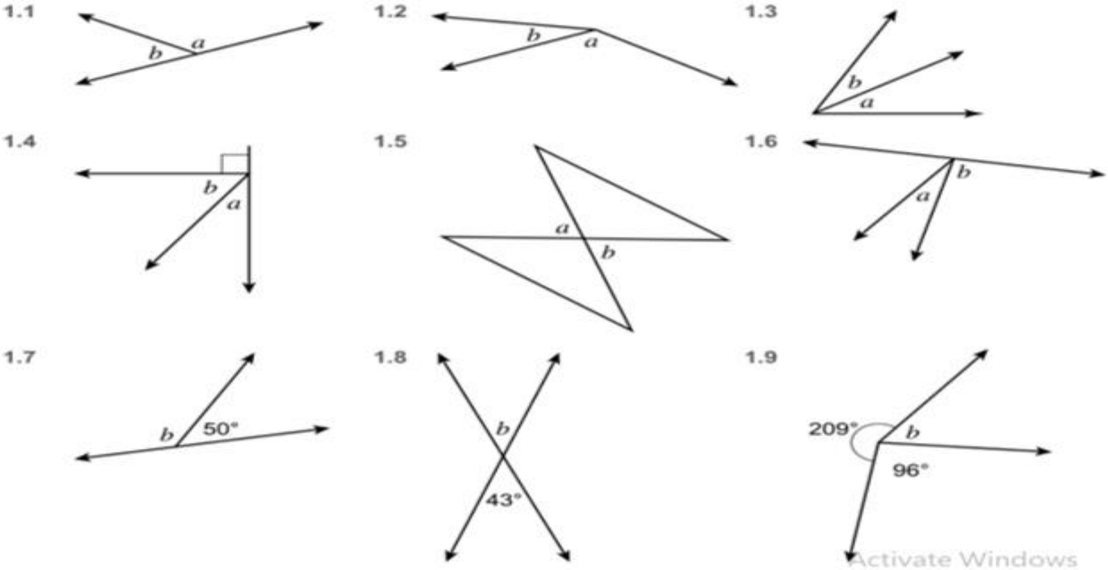
- Session introduction: 10 minutes.
- Pre-test/Know your basics, written/oral: 10 minutes.
- Pre-test/ Know your basics, written/oral feedback: 10 minutes.
- Prior knowledge identification: 10 minutes.
- Angle Relationships and Solving Geometric Problems: 50 minutes.
- Discussions around identifying misconception and teaching strategies development: 25 minutes.
- Final remarks: 5 minutes.

The activities and worksheets to be used in the workshop (maximum 8 pages)

Workshops need to be hands-on sessions where participants are actively involved in doing the activities that you provide. Usually these activities will be done in groups, consisting of 3–5 participants. There should also be ample time for discussions (approximately 25% of your time is suggested). If you have used ideas from other sources, it is essential that you acknowledge these sources.

Know your basics

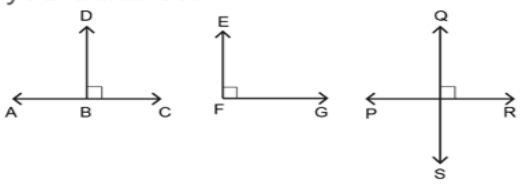
1. Name the relationship between the angles shown using the following words: complementary; linear; adjacent; vertically opposite; angles around a point.



Pearson

Activity

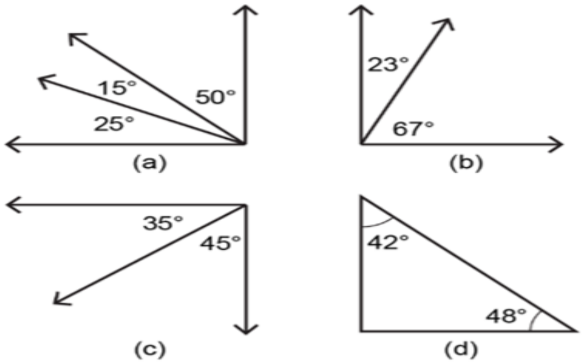
1. Identify which of the sets of lines in the following figures is perpendicular and explain why you think so.



2. Draw your own pair of complementary angles.

Pearson

3. In groups, identify which set of angles are complementary, and explain why you think so.



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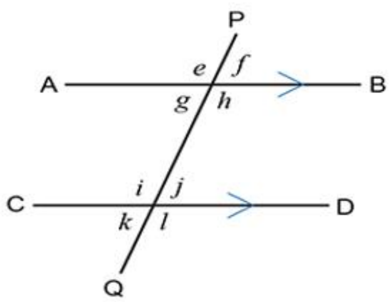
ACTIVITY

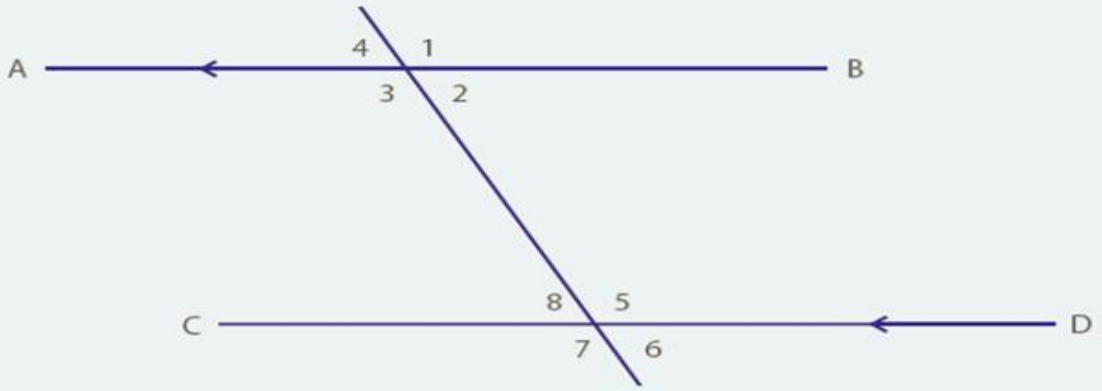
In groups, identify which set of angles are corresponding, alternate and co-interior angles, and explain why you think so.



Pearson

Example

In the diagram, AB||CD the two lines || indicate that AB and CD are parallel to each other. Which angles are equal to each other? Give a reason for each of your answers.



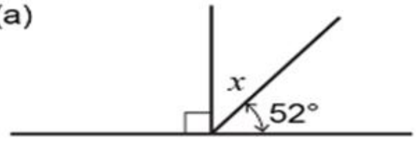
Activity

1. Draw three different pairs of complementary angles and label them.

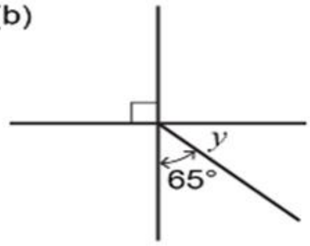
2. Draw three different pairs of supplementary angles and label them.

3. Work out the value of the unknown in each of the following figures, giving reasons for your answers.

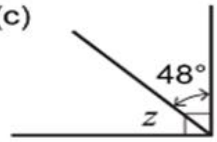
(a)



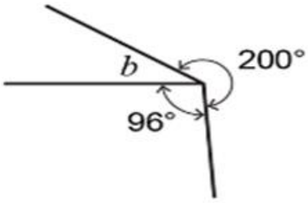
(b)



(c)

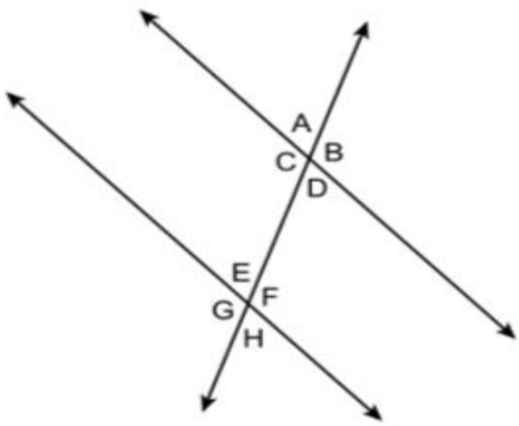


(d)



Activity

2.



Name the angle relationships between the following angles:

2.1 $\hat{A}$ and $\hat{C}$

2.2 $\hat{D}$ and $\hat{E}$

2.3 $\hat{G}$ and $\hat{B}$

2.4 $\hat{C}$ and $\hat{F}$

2.5 $\hat{F}$ and $\hat{D}$

2.6 $\hat{A}$ and $\hat{E}$

2.7 $\hat{C}$ and $\hat{E}$

2.8 $\hat{B}$ and $\hat{F}$

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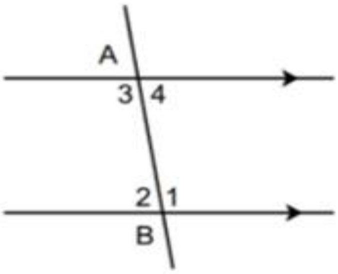
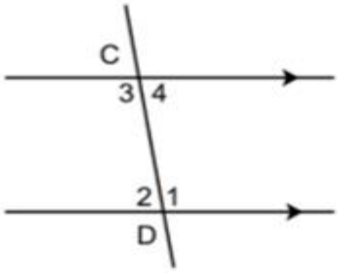
3. Study the diagrams and find the missing angles, given that:

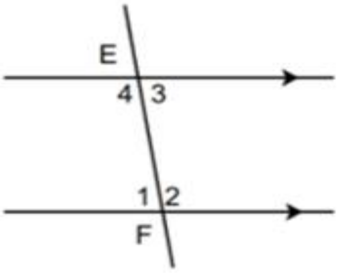
3.1 $\hat{A}_3 = 99°$



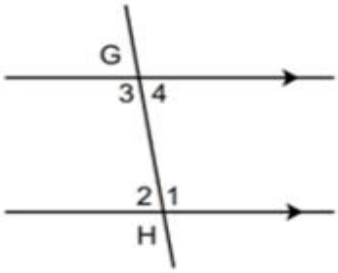
3.2 $\hat{C}_3 = 84°$



3.3 $\hat{F}_1 = 126°$



3.4 $\hat{H}_2 = 85°$



Facts, fluency and fun: Exciting ways to get on top of basics in Intermediate and Senior Phase

Baatseba Seage Mamaro & Patrick Iroanya

OLICO Maths Education

In this workshop educators will be introduced to the Two-Minute Tango app. This app has been designed and tested in South Africa by OLICO mathematics education and has proved to be both engaging and successful in improving learners' fluency in the basic operations. It is available for free on the google play store (https://play.google.com/store/apps/details?id=com.eucleia.olicomaths&gl=ZA) and, for those without access to a smart device a paper-based version is available. (2-minute tango)

OLICO mathematics education designed the Two-Minute Tango programme in response to our experience that lack of fluency in the four basic operations was hampering the Intermediate Phase and Senior Phase learners in our afterschool mathematics programmes from accessing their grade level mathematics.

In this workshop participants will get hands-on experience of both the paper-based and app-based versions of the 2-minute tango. We will discuss ways in which they can be used in class or at home.

(Note: the app is currently only available on android. Those without an android device can log on to Course: Two-Minute Tango for a web-based version of the two-minute tango)

Intended audience:

This workshop is intended for teachers who intend to improve their learners fluency skills particularly in the Intermediate and Senior phase programs.

Duration:

The workshop will be 1 hour long.

Maximum number of participants:

There is no specific limit of the number of participants since the workshop will be online.

Motivation for the workshop

It is estimated that learners from the lower quintile schools are 3 – 4 years behind their expected grade level in mathematics (Spaull and Kotze, 2015). In our work providing after-school support in mathematics to learners in grades 4 – 9 we have noted that a lack of fluency with basic addition, subtraction, multiplication and division significantly hamper our learners' ability to access the mathematics they are trying to learn. We noted that many learners, even those in early high school would still revert to calculating by unit counting using their fingers.

In response to these concerns, OLICO Maths education has developed the *2-minute tango*, focused on building fluency in the 4 operations. This has proved successful in our work with learners. This workshop is motivated by our desire to share what has worked well with our learners with different organizations and teachers so that they have the option to use the material in their study plan for their learners.

Description of content

2-minute tango: These are a carefully sequenced set of fluency exercises that focuses specifically on the four operations in mathematics i.e. $+$; $-$;$\div$ and $\times$. The 2-minute tango exercises are divided into 3 sections. The first focuses specifically on addition and subtraction and aims to get learners able to fluently add and subtract up to 2-digit numbers mentally. The second section focuses on building learners' fluency in single digit multiplication and division facts along with further multiplication and division calculations that can be derived from these. The third section focuses on instant recall of times tables.

These exercises can either be practiced on an android smart device, online or on an offline worksheet. This exercise is very impactful on learners as it helps them to acquire the necessary fluency skills.

What will be done in the workshop?

- Introduction by presenters: $\pm$10 minutes
- Trying out the 2-minute tango fluency exercises on addition and subtraction with discussion of accompanying instructional material: $\pm$15 minutes
- Trying out the 2-minute tango fluency exercises on multiplication and division with discussion of accompanying instructional material: $\pm$15 minutes
- Trying out the 2-minute tango times table challenge: $\pm$10 minutes
- Wrap-up discussion: $\pm$10 minutes

Activities and materials provided

There will be two versions of the fluency activities. The offline and online versions.

Offline Activities:

Participants are required to download the offline versions of the activities prior to the workshop from the following links:

2-minute tango activities:

The offline activities can be downloaded here: 2-minute tango

Online Activities.

Participants are required to create an OLICO account and download the 2-minute tango app on their android smartphone.

Create an OLICO account here: New account
Download the 2-minute tango app here: https://play.google.com/store/apps/details?id=com.eucleia.olicomaths&gl=ZA

Access the web-version of the 2-minute tango here: Course: Two-Minute Tango

Technological requirements

Each participant must have a computer and/or a smart device and good internet connection.

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(Un)common factors and multiples of fractions

Connie Skelton

Iconic Maths

I included a revision exercise for LCM and HCF in the Iconic Mental Mathematics Grade 9 workbook in the form of the LCM and HCF of fractions with a formula for calculating each. The exercise was designed as an extension exercise, substitution as well as revision of LCMs and HCFs. I also included the exercise as one of the Casio Calculator competition questions at the AMESA National Congress 2017. The latter sparked a wonderful debate with an esteemed colleague at that AMESA Congress about:

- *what the LCM and HCF of rational numbers means*
- *how that impacts on the subset that is included in the Senior Phase CAPS for Grade 7 and 8, and*
- *whether it is relevant because it is not included in CAPS at all.*

This workshop investigates some practical and theoretical aspects of the HCF and LCM. It is not directly related to CAPS as such but uses the background knowledge. It should stimulate and challenge teachers' thinking and concepts of this aspect of Mathematics and perhaps give some ideas for discovery exercises for the Senior Phase.

Target audience: Senior Phase and any other interested delegates
Duration: 1-hour hands-on workshop (Power Point)
Maximum number of participants: *n*
Requirements: a sheet or two of recycled paper to cut up, place to write, two pens or highlighters of different colours, a pair of scissors, a ruler, calculators (if necessary).

Motivation for the workshop:

Often, we are challenged to think more deeply and consider the fundamental concepts of an aspect of Maths by delving into an area that branches from the known and the familiar. This workshop is a hands-on tour of factors and multiples, with some practical considerations. As the calculator technology advances, it is necessary to consider other ways of examining the LCM and HCF sections of the curriculum.

Description of the content of workshop:

Introduction		5 minutes
Activity 1	Dividing into equal pieces	10 minutes
Activity 2	Dividing specific lengths into equal pieces	5 minutes
Anthyphairesis and the highest common factor		10 minutes
Discussion		5 minutes
Why is it relevant?		5 minutes
General Properties of HCF and LCM		10 minutes
That exercise that caused the debate!		5 minutes
Conclusion		5 minutes

Introduction

We are going to start with two practical examples of what the Highest Common Factor means. I hope that you will all participate as it is an easy and fun way to understand the concept of something we normally only do numerically. Please also briefly fill in your Hypothesis (what you think will happen) and the Conclusion at the end. Some or all of the Discussion questions can be used in the classroom situation if you would like to convert this into a lesson. The questions are designed to encourage learners to verbalise their Mathematics.

Activity 1 Dividing into equal pieces

Aim:
Is it possible to divide two different strips of paper into equal smaller pieces so that there is no left over paper from either strip?

Hypothesis:
What do you think will happen?

Materials:
A sheet or two of recycled paper to cut up, place to write, two pens or highlighters of different colours, a pair of scissors, a ruler

Method:

1 Use the pieces of recycled paper to cut two strips about 2 cm wide.
2 Make the one strip shorter than the other.
3 Draw a line down both sides of one of the pieces of paper with one of the highlighters or pens and with the other pen or highlighter down both sides of the other strip.
4 Place the shorter strip above the longer one so that one end lines up. Cut off the excess (C) of the longer one.
5 Place the excess (C) one on top of the other two and cut off the excess again.
6 Continue taking the shorter pieces and putting them on top of the longer ones and cutting off the excess.
7 You should get to a point where all the pieces are the same size.

Results:

What did you see?

Discussion:

1 What do you know for sure? (Asking questions)
2 What are you trying to find out? (Asking questions)
3 Are there any special rules or conditions that you need to be aware of? (Asking questions)
4 What does this situation remind you of? (Maths-to-self connection)
5 Is it related to anything you have seen anywhere? (Maths-to-world connection)
6 What is the main idea from Maths that is happening here?
7 Is this like some other Maths ideas? (Maths-to-Maths connections)

Conclusion:
Did the experiment confirm your hypothesis? Why or why not?

Activity 2 Dividing specific lengths into equal pieces

Aim:
Is it possible to divide two different strips of paper into equal smaller pieces so that there is no left over paper from either strip?

Hypothesis:
What do you think will happen?

Materials:
a sheet or two of recycled paper to cut up, place to write, two pens or highlighters of different colours, a pair of scissors, a ruler

Method:
Use the pieces of recycled paper to cut two strips about 2 cm wide.

1 Make the strips 20 cm and 12 cm long respectively.

3-7 Follow points 3 to 7 in Activity 1.

8 You should get to a point where all the pieces are the same size. Measure their lengths.

Results:
What is the length of all the equal pieces?

Discussion:

1 What do you know for sure?

2 What are you trying to find out?

3 What does this situation remind you of? (Maths-to-self connection)

4 What is the main idea from Maths that is happening here?

5 Is this like some other math ideas? (Maths-to-maths connections)

Conclusion

Did the experiment confirm your hypothesis? Why or why not?

Anthyphairesis and the highest common factor

Anthyphairesis (antha-fair-isis) of two different quantities is a method of repeatedly removing. It consists of subtracting the smaller of the two quantities from the larger one. The process continues until the quantities are constant. The result is unit U that is common to both original segments. Each segment contains the common unit a certain number of times: A = *n*U and B = *m*U.

Let's consider 12 and 20, the two lengths of the strips in Activity 2 above:

$12 * 8 \;\; * 4 * \mathbf{4} * 0 \;\; * 4 * 0 * \ldots$

You will see that the two strips in Activity 2 divided into 4 cm pieces.

Why is this significant?

4 is the Highest Common Factor of 12 and 20.

Anthyphairesis comparison is an ancient procedure which goes back to Euclid's time. Loosely translated Anthyphairesis means alternated subtraction. It works for whole numbers and fractions as long as they are not incommensurate (like the side and diagonal of a square). They must be able to form a rational number.

Example 1

Let's apply Anthyphairesis comparison to the following two lengths:

$2\frac{2}{3}$ and $10\frac{2}{5}$

1 Convert both mixed numbers to improper fractions:

$2\frac{2}{3} = \frac{8}{3};\; 10\frac{2}{5} = \frac{52}{5}$

2 You might find it easier to subtract using a common denominator:

$\frac{40}{15}; \frac{106}{15}$

3 $\frac{40}{15} * \frac{106}{15} * \frac{66}{15} * \frac{40}{15} * \frac{26}{15} * \frac{14}{15} * \frac{12}{15} * \mathbf{\frac{2}{15}} * \frac{10}{15} * \frac{8}{15} * \frac{2}{15} * \frac{6}{15} * \frac{4}{15} * \frac{2}{15} * \mathbf{\frac{2}{15}} * 0$

From this we see that $\frac{2}{15}$ is the HCF of $2\frac{2}{3}$ and $10\frac{2}{5}$. This means that $\frac{2}{15}$ will divide into each of the values by a whole number value, *m* or *n*:

$$\frac{8}{3} \div \frac{2}{15} = \frac{8}{3} \times \frac{15}{2} = 4 \times 5 = 20$$

$$\frac{52}{5} \div \frac{2}{15} = \frac{52}{5} \times \frac{15}{2} = 26 \times 3 = 78$$

Example 2

$15\frac{3}{4}$ and $12\frac{5}{6}$

1 Convert both mixed numbers to improper fractions:

$$15\frac{3}{4} = \frac{63}{4};\ 12\frac{5}{6} = \frac{77}{6}$$

2 You might find it easier to subtract using a common denominator: $\frac{189}{12}$; $\frac{154}{12}$

3 $\frac{189}{12} * \frac{154}{12} * \frac{35}{12} * \frac{119}{12} * \frac{84}{12} * \frac{35}{12} * \frac{49}{12} * \frac{14}{12} * \frac{35}{12} * \frac{19}{12} * \frac{16}{12} * \frac{3}{12} * \frac{13}{12} * \frac{10}{12} * \frac{3}{12} * \frac{7}{12} * \frac{4}{12} * \frac{3}{12} * \frac{1}{12} * \frac{2}{12} * \frac{1}{12} * \frac{\mathbf{1}}{\mathbf{12}} * \frac{0}{12} * \frac{1}{12}$

Discussion

You will notice that the values of the HCF of fractions $\frac{N}{D}$ can be simplified to:

HCF $\frac{N}{D} = \frac{HCF(n_1;n_2)}{LCM(d_1;d_2)}$

Anthyphairesis can only be used to compare two homogeneous quantities and the equation above gives us a method of calculating the HCF of two or more quantities.

For those that are interested, here is a proof, but there are other proofs available too.

Proof:

Let the fractions be $\frac{n_1}{d_1}$ and $\frac{n_2}{d_2}$. Let the HCF be $\frac{N}{D}$.

The HCF is the highest possible number that divides both fractions without a remainder. We need to maximize *N* and minimize *D*.

$\frac{n_1}{d_1} \div \frac{N}{D} = \frac{n_1}{d_1} \times \frac{D}{N}$ should have no remainder (be a whole number)

$\frac{n_2}{d_2} \div \frac{N}{D} = \frac{n_2}{d_2} \times \frac{D}{N}$ should have no remainder (be a whole number)

=> N must be the HCF of n_1 and n_2 to ensure N divides both and is as large as possible.

=> D must be the LCM of d_1 and d_2 to ensure that it is a multiple of d_1 and d_2 and is as small as possible.

This brings us to the definition of the LCM of fractions as the smallest possible number that all the fractions under consideration can divide into by a whole number.

Technically, a multiple can be negative. There are negative factors and multiples of numbers, for example if the question was:

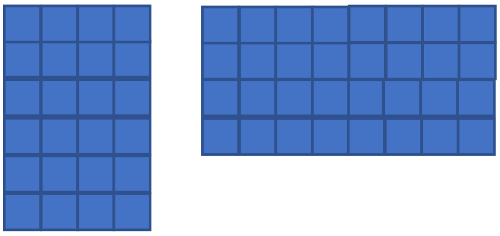
How many factors does 6 have? The answer would be 8: (-6, -3, -2, -1, 1, 2, 3, 6).
Strictly speaking, we should say: “How many positive factors does 6 have?”

Why is it relevant?

1 Exploring Mathematics deepens our understanding of other concepts, like the limited subset of the LCM and HCF that is required in CAPS and has wider applications in problem-solving in general.

2 Practically speaking it gives us a way of dividing homogeneous lengths (and areas, volumes, too) into equal-sized portions. Think of technology, fabric, yarn, wood or even fence posts. Can you think of other applications in the Mathematics curriculum itself?

In terms of areas:



a What is the largest square that can divide into both the rectangles without remainders?

b What is the largest rectangle that can divide into both rectangles without remainders?

c Show how the HCF of the sides could be used to find the largest squares.

3 This gives teachers a different way to explore this section in tests and exams as calculators advance and can calculate LCMs and HCFs.

General properties OF LCM and HCF

1 The product of the LCM and HCF of any ***two*** give natural numbers is equivalent to the product of the given numbers.

2 The HCF of co-prime numbers is 1. (Co-prime numbers have no common factors except 1). Therefore, the LCM of co-prime numbers is the product of the numbers.

3 HCF and LCM of fractions

$$\text{HCF } \frac{N}{D} = \frac{HCF(n_1;n_2;n_3;\dots)}{LCM(d_1;d_2;d_3;\dots)} = \frac{HCF\ of\ the\ numerators}{LCM\ of\ the\ denominators}$$

LCM $\frac{N_l}{D_l} = \frac{HCF(n_1;n_2;\, n_3;\ldots)}{LCM(d_1;d_2;\, d_3;\ldots)} = \frac{LCM\ of\ the\ numerators}{HCF\ of\ the\ denominators}$

4 The HCF of any two or more numbers is never greater than any of the given numbers.

5 The LCM of any two or more numbers is never smaller than any of the given numbers.

The exercise that caused the debate!
LCM and HCF [From *Iconic Mental Maths Grade 9*]

HCF of fractions = $\frac{\textbf{HCF of numerators}}{\textbf{LCM of denominators}}$

LCM of fractions = $\frac{\textbf{LCM of numerators}}{\textbf{HCF of denominators}}$

1 HCF of $\frac{2}{9}$ and $\frac{8}{15} = \frac{2}{45}$

LCM of $\frac{2}{27}$ and $\frac{8}{9}$ = ______

2 HCF of $\frac{7}{3}$ and $\frac{22}{15}$ = ______

LCM of $\frac{7}{3}$ and $\frac{22}{15}$ = ______

3 HCF of $\frac{12}{11}$ and $\frac{4}{15}$ = ______

LCM of $\frac{12}{11}$ and $\frac{4}{15}$ = ______

4 HCF of $\frac{8xy^2}{12x^2y}$ and $\frac{14xy^3}{42y^2}$ = ______

LCM of $\frac{8xy^2}{12x^2y}$ and $\frac{14xy^3}{42y^2}$ = ______

[From *Iconic Mental Maths Grade 9*]

Conclusion

The HCF and LCM of fractions or rational numbers do exist and do have meaning in terms of our definitions of the HCF and LCM in our limited subset of only the whole numbers in our curriculum. There is a wider use for these values when one considers measurements.

One of the most wonderful aspects of attending the AMESA National Congresses is that there is always the possibility of sparking a discussion with a colleague that takes you out of your general Mathematical languor and makes you reconsider the real meaning of the smaller subset that we cover in our curriculum. These experiences are precious and unpredictable, and we necessarily will not always know the answers or always be right. It is in the debate or discussion that our understanding is broadened and makes us love this amazing subject called Mathematics. I love the use of the technology to have the privilege to participate in this AMESA Conference and I cannot wait for the time we meet again at an AMESA Congress somewhere.

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Two negatives make a positive. Navigating integer errors

Yvonne Sanders, Iresha Ratnayake & Vasantha Moodley

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Introduction

Negative numbers are difficult to learn but also difficult to teach. Errors with negative numbers influence learner performance in algebra and it is therefore essential that learners' difficulties with negative numbers are addressed. In our presentation we will discuss some of the most common errors that learners make with negative numbers and we show how we address these errors in worksheets developed by the Wits Maths Connect (WMCS) Project, a research and development project that aims to improve the teaching and learning of mathematics in secondary schools in South Africa. We strongly believe that errors provide opportunities to learn from and that teacher knowledge of these errors can enhance the teaching and learning process.

The four types of errors in integers

Four types of integer errors have been identified in local and international research. We describe each of these briefly and provide some examples to illustrate them.

Right to left reasoning is where learners read the expression from right to left. This usually occurs when the expression contains a larger number subtracted from a smaller number, e.g. $7-10$. Here a learner might read it from right to left and say "10 subtract 7 gives me 3".

2) ***Bracket reasoning*** occurs when learners insert imaginary or explicit brackets into an expression. Sometimes this inadvertently changes the question. For example, in $6-10-8$, learners might insert imaginary brackets around $10-8$ and get 2 and then obtain $6-2=4$. Figure 1 shows an example of a possible way that learners would work on this

$6-10-8 = 6-(10-8) = 6-2=4$

Figure 1:

3) ***Ignoring the leading negative*** is an error learner may make when the first number is negative. For example, with $-7-5$, learners may ignore the leading negative and operate on the $7-5$ first after which they "re-attach" the negative symbol to get a final answer of -2.

4) ***Misusing the sign rules*** is an error where learners apply the signs rule for multiplication to an addition or subtraction question. A statement often heard in the classroom is "a negative and a negative make a positive".

Learners might incorrectly apply this to: $-3-10$ and respond with $+13$ because the "two negatives make a positive".

Strategies to address the errors

After identifying the common errors one major challenge teachers face is how to help learners to overcome them. In our worksheets we used a variety of tasks that consist of different questioning styles as well as examples and counter examples to focus on errors. Figures 2 – 5 demonstrate how we use different tasks to address all four errors. Figure 2 addresses right to left reasoning by requiring learners to write out number sentences. Bracket reasoning is addressed in figure 3 as it contains questions that we know learners would typically insert brackets. In figure 4 the leading negative is addressed by questions that have missing values whereas in figure 5 learners are confronted with contrasting examples using both products and sums in order to highlight the difference between the signs rule for multiplication and addition.

Write as a number sentence, then calculate the answer:

a) Five subtract thirteen e.g. $5 - 13 =$

b) Six subtract ten

Figure 2:Right to left reasoning

Calculate the following:

a) $12 - 10 - 4 =$

b) $3 - 4 + 5 =$

c) $4 - 3 + 3 =$

d) $7 - 5 - 6 =$

Figure 3: Bracket reasoning

Fill in the missing numbers to make each statement true:

a) $-15 + \_\_ = -1$

b) $-15 - \_\_ = -1$

c) $-15 - \_\_ = 20$

d) $-15 - \_\_ = -20$

e) $\_\_ - 20 = -1$

Figure 4: Leading negative

Choose *positive* or *negative* to make these statements true:

a) The product of 3 negative numbers is positive/negative

b) The sum of 3 negative numbers is positive/negative

c) The sum of 2 negative numbers is positive/negative

d) The product of 2 negative numbers is positive/negative

Figure 5: Misuse of the signs rule

In the presentation we will provide more examples from worksheets we have developed to show how we have used different types of tasks to address the different errors. We hope that sharing some of our worksheets and explaining why we have varied the questioning style, language used, and the examples chosen to provide some strategies to address identified learner errors in integers.

How to use Polya's four-step approach to Problem Solving in Euclidean Geometry

Yudeisy Cudina Guerrero; Pedro Reinaldo Pérez Campbell; Phillip k. Dikgomo
Basic Education Department

One of the primary reason's learners have trouble with problem solving is that there is no single procedure that works all the time, each problem is slightly different. Also, problem solving requires practical knowledge about the specific situation. If you misunderstand either the problem or the underlying situation you may make mistakes or incorrect assumptions.

In 1945 George Polya published the book How to solve it which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. In this book he identifies four basic principles of problem solving.

One of our main goals for this paper is to give teachers Further Education & Training some methodological ideas in how to solve Geometry problem using the Polya's method.

Polya's four-step approach to Problem Solving

To begin this task, we will discuss a framework for thinking about problem solving: Polya's four-step approach to problem solving.

1) Understand the problem

- 1- What sort of a problem is it?
- 2. What is being asked?
- 3. What do the terms mean?
- 4. Is there enough information or is more information needed?
- 5. What is known or unknown?
- 6. Have I solved a similar problem?

2) Devise a plan

1. Draw pictures, graph or diagram
2. What concepts are the given data related to?
3. What formula relates the data to the unknown?
4. Will there be any sub problem that when solving it helps me to solve the problem?

3) Carry out the plan

1. Review of the steps if they are correct or not
2. Make sure the calculations made are correct

4) Look back

1. Did you answer the question?
2. Is your result reasonable?
3. Double check to make sure that all of the conditions related to the problem are satisfied.
4. Double check any computations involved in finding your solution.

To continue with this paper, we would like to remind you, what is problem solving; and as you know, it is the process or act of finding a solution to a problem. Moreover, all of us know what problem means, that is, a question raised for inquiry, consideration, or solution. Therefore, a geometry exercise is also considered a geometric problem. Now we will show you through an example for learners Further Education & Training, how through a geometry exercise we can use the Polya's four-step approach.

With this simple example that includes properties of the isosceles triangle and angles in the circumference, we intend to demonstrate the usefulness of the Polya’s four -step approach and how through these we achieve that learners can solve geometric problems independently. And how through these we achieve a set of questions that the students must formulate allowing them to connect with the concepts, formulas, properties of figures that they already know. And thus, also achieve improve strategic and logical thinking in our learners.

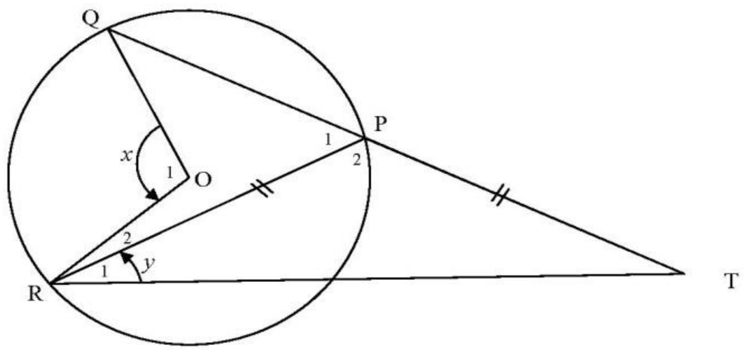
Example:

In the figure along-side, O is the center of the circle and PT = PR

Let $\hat{R}_1 = y$ and $\hat{O}_1 = x$

a) Express x in terms of y

b) Hence calculate y if $x = 120^o$



In the first step, the learners identify the type of problem they are going to solve, and in this case it is a geometric problem, moreover in the second step, the learners should be able to relate concepts to the given data, such as circle and triangle, as well as, formulas and theorems that relate the given data to the unknown, after that the learners should carry out the plan, they must ensure that the calculations performed are correct, and finally they must to recheck to make sure all conditions related to the problem are met.

This paper aims to put in the hands of teachers a guide on how through the Polya’s four-step approach we can solve, not only problems, but any type of mathematical exercises, always keeping in mind that our students are the center of the teaching-learning process of the mathematics.

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Using decompression and re-representation in constructing examples for exponents

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Background

Exponents are an important part of school mathematics. Exponential notation helps in writing very large and very small measurements to significant digits. While exponential notation is a numerical representation, exponents arise very often in algebra. Like other topics in mathematics, learners need to gain an understanding of the laws of exponents and not merely apply them mechanically. Anticipating how learners would deal with the topic and responding to their questions is an important consideration in designing mathematical tasks. In exponents, for instance, a legitimate learner question could be why is $t^2 \times t^2 = t^4$ but $t^2 + t^2 = 2t^2$and not t^4 or $2t^4$. How do we justify this to the learner? Such questions make us wonder what is visible to learners in the exponential notation and what is not?

In this contribution, we propose two key ideas of *decompression* and *re-representation* and we show how they can be used when teaching exponents. By decompression, we mean seeing the structure of the exponent by expansion. For example, $t^2 \times t^2$ can be written as $t \times t \times t \times t$. By re-representation, we mean presenting a power in a different but equivalent exponential form. For example, $(2x)^{-2}$can be written as $\frac{1}{4x^2}$ or $(4x)^{-1}$. We suggest that several learner difficulties with exponents can be addressed if they are given opportunities to decompress and represent exponential notation in different ways.

To show how *decompression* and *re-representation* can be used when teaching exponents to Grade 8 and 9 learners[2], we will use mathematical tasks from

[2] The selected examples might be suitable for Grade 10 learners.

worksheets that have been developed by the Wits Maths Connect Secondary (WMCS) project. Below, we share core principles on how the worksheets were developed in the project, whereby the nature of questions in the worksheets (we believe) provides learners with opportunities to decompress and represent exponential notation in different ways.

The WMCS project has been developing booklets of worksheets for learners to practise specific mathematical procedures and to reinforce important concepts. The purpose of these worksheets is to expose learners to different types of questions which are designed to reveal learners' mathematical errors and misconceptions. The development of the worksheets begins with identifying the content that Grade 8 and 9 learners need to know and do and move to identify which specific sub-topic the learners need to understand and in which way. On the WMCS website[3], there are worksheets for all the laws of exponents, but for our purposes, we will draw example sets on the worksheet focusing on the power law of exponents.

Example sets on the power law of exponents

The power law states that, when raising a power to a further power, we multiply the exponents, that is, $(a^m)^n = (a^{m \times n})$, and for this presentation m and n are integers. How do we then justify the multiplication of exponents in this law? We know that an expression can be represented in multiple ways. For instance, $(a^m)^n$ can be expanded as $(a \times a \times a \ldots m\ times)^n$ or $(a \times a \times a \ldots n\ times)^m$ or $(a \times a \times a \ldots mn\ times)$. An awareness of these multiple ways of representing a power or re-representation provides flexibility to the learner. It reinforces the expansion of the expression in different ways, which can serve as the rationale for the power law. In what follows, we discuss three example sets (questions) to demonstrate how learners' attention can be drawn to re-representation and decompression.

[3] For more details, visit https://www.witsmathsconnectsecondary.co.za/resources

Example Set 1:

Wanda and Shikha show that $(a^{-1})^{-1} = a$ using different methods that are both correct.

Wanda's method: $(a^{-1})^{-1} = a^{-1\times-1} = a^1$ or a.

Shikha's method: $(a^{-1})^{-1} = \frac{1}{a^{-1}} = a$ or a^1

a) Describe each method in words. Focus on what is different about the methods.

b) Which method do you prefer? Why?

c) Simplify the two examples below using the method you prefer. Give answers with positive exponents:

   i) $(5^{-1})^{-4}$

   ii) $(2^{-5})^{-2}$

In this example set, the purpose is to draw learners' attention to two different ways of justifying the equivalence of $(a^{-1})^{-1}$ and a. The learners are shown two different ways of representing $(a^{-1})^{-1}$, first using the power law of exponents and second using the relation between positive and negative exponents. You will see that the learners are not asked to solve a question in the beginning but are expected to "see" the two methods and identify the differences. This question emphasizes the idea of re-representation where different mathematical ideas can be used to justify a procedure. Putting emphasis in this idea of re-representation offers an opportunity for learners to connect different mathematical ideas in a coherent way. Then, the learners can practice simplifying powers with different types of questions such as question *c* in this example set.

Example Set 2:

Look at the powers in the list below. Group the powers which are the same. $(-y)^{-4}$; $\left(\frac{1}{2}(y)^{-2}\right)^2$; $((-y)^{-2})^2$; $(4(2y)^{-1})^2$; $(2^{-1}y^{-1})^2$; y^{-4}

This example set also addresses the key idea of re-representation in a different way. Learners are required to group the powers that are the same. In order to identify the same power, for example in this case, the learners can pick one power from the list, which can be in its simplest form, like $(-y)^{-4}$, and look for another power that can yield the same power when multiplying the exponents while the base is $-y$. From the rest of the powers in the list, none is the same, but to conclude this, the learners are required to re-represent each power to see if it can match with other powers. For example, $(\frac{1}{2}(y)^{-2})^2$ can be represented in several ways including $\frac{1}{4}y^{-4}$ and $\frac{1}{4y^4}$, which do not match with other powers in the list.

Example Set 3:

Match the expansions with the powers.

Expansion	**Power**

a) $2y \times 2y$ b) $\frac{2}{y} \times \frac{2}{y}$ c) $y \times y \times y \times y$ d) $\frac{1}{y^{-2}} \times \frac{1}{y^{-2}} \times \frac{1}{y^{-2}} \times \frac{1}{y^{-2}}$	i) 2^2y^{-2} ii) $(y^2)^2$ iii) 2^2y^2 iv) $(y^{-2})^4$ v) $(y^4)^1$

In this example set, learners are expected to match the power and its expanded form. The coefficient (2) and variable (y) are kept being the same, to draw learners' attention to the different ways in which a power can be expanded. It is the expanded form that helps the learners in justifying the laws of exponents. In this case, the learners can justify the power law using the expanded notation. Let us pause to think about the different ways in which we can expand 2^2y^{-2}. Here are some possible ways of writing the power: $2 \times 2 \times \frac{1}{y} \times \frac{1}{y}$, $\frac{2}{y} \times \frac{2}{y}$, $(\frac{y}{2})^{-2}, 4y^{-2}$, etc. When decompressing the exponential notation, we see that the power can be expanded in different ways depending on what we want to draw learners' attention to.

Conclusion

From our experience, it can be noted that the nature of questions themselves can enable teachers to draw learners' attention to the key ideas of *decompression* and *re-representation* when teaching exponents. We find these ideas to be useful in teaching other laws of exponents as well.

Acknowledgement

This research and development work is supported financially by the FirstRand Foundation, the Department of Science and Innovation and the National Research Foundation.

Conceptualization of Multiplication in the Foundation phase

Nkambule Adam

Umpopoli Primary School

Target : Foundation Phase Educator

Duration : 20 Minutes

Multiplication awards learners with opportunities to think additively. For learners to be able to multiply they must first master counting and addition conceptual since there is a huge relationship between addition and multiplication. Park and Nunes (2001) emphasis that multiplication is grounded on understanding repeated addition. And Park and Nunes further proposed that repeated addition is only a calculation procedure and that the understanding of multiplication has its roots in the schema of correspondence. According Clark and Kami (1996) to multiply is the same as doing repeated addition. For an example 3 X 4 = 12 hence with repeated addition it will be 3+3+3 + 3 = 12 or 4 + 4 + 4 = 12.

Strategy 1

Adding it up

When learners solving 8 x 5 can illustrate their mental process as 8+8 = 16, 16 +8 = 24, 24+ 8 = 32, 32 +8 = 40. Learners will be using repeated addition to get to the answer thus we know from definition that multiplication is repeated addition for example 4 X 3 = 12 using repeated addition the illustration will be 4 +4+4 = 12.

Strategy 2

Skip counting

Learners will be using their fingers to count yet this process require maximum concentration. When a learner is given 10 X 7 with skip counting the learners will use the following method to get to the answer 10, 20, 30, 40, 40, 60, 70. Thus the answer will be 10 X 7 = 70.

Strategy 3

Using the known fact:

Learners can use multiplication using the known, for an example 9 X 6 the learners can use known fact by 9+ 9 = 18, the they are 2 (9s) in 18 hence the representation can be as follows: 18 + 18 +18 = 54 thus 9 X 6 = 54.

Strategy 4

Counting in Steps

At early grades which is reception learners must be taught in multiplying by counting in steps. For example

5 X 6 =

5, 10, 15, 20, 25, 30 hence is 6 steps of 5.

5 X 6 = 30

Or

5 X 6 =

6, 12, 18, 24, 30 hence is 5 steps of 6.

5 X 6 = 30

Learners can count in steps using concrete object or counters.

Strategy 5

Arrays

Arrays are set of objects arranged into rows and columns.

7 x 3 =

The triangles in the box are examples of how arrays can be used for multiplication, the learners will get the answer by counting all the triangles in that are into rows and columns.

7 x 3 = 21.

Strategy 6

Partitioning

Partitioning method work best when learners start multiplying bigger numbers. The partitioning is breaking numbers into the hundreds, the tens and the units. The partitioning will be first break down the numbers so that they can work with simple numbers using they pre-knowledge of breaking down see the below example: for example:

a) 57 X 3 (50 + 7) X 3

(50 X 3) + (7 X 3)

150 + 21

100 +50 +20 +1

171

57 x 3 = 171

b) 64 x 4 =

(60 + 4) X 4

(60 X 4) + (4 X 4)

240 + 16

200 +40 + 10 +6

256

64 X 4 = 256

13 x 7

(10 + 3) X 7

(10 X 7) + (3 X 7)

70 + 21

70 + 20+ 1

91

Strategy 7

Grids

This is a complex multiplication strategy it can be used when learners develop understanding they can use grid to multiply it involve knowledge of partitioning learnt.

For example

58 X 3 =

x	50	X	8	
3	150		24	174

Step 1 is portioning 58 = 50 + 8

Step 2 is Multiplying 50 by 3 which is =150

Step 3 is Multiplying 8 x 3 which is = 24

Step 4 adding 150 and 24 which is = 174 which is the product of 58 and 3 or

Strategy 8

Vertical format

This is a strategy that is used by most learners yet the Curriculum Policy Statement does not recommended it for early grade but learners can use it to check they answer which they are found while using any of the above strategies that are recommended by Curriculum Policy Statement. Below is the example of multiplication using vertical format:

$$\begin{array}{r} 23 \\ \times\quad 3 \\ \hline 69 \end{array}$$

I hope my contribution will optimize the teaching and learning of multiplication in the foundation phase and mathematics in general.

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Measurement: Telling the time in the Foundation Phase

Matsie Sebeela

St Mary's Primary School

Target audience: Foundation Phase Grade R-3

Duration: 1 hour

Maximum number of participants: 30 participants

Motivation

Measurement is one of the most useful educational skills that should be taught in Mathematics, especially time as it allows children to manage themselves and their activities effectively, they also learn to be responsible. Children learn to value time and use it wisely once they come to understand that once it has passed, it can never be brought back. They learn important skills of determining whether they are running late or still have time to complete an activity. These are important skills that are needed even in adulthood so that one can run their lives efficiently. Being able to tell time helps learners to develop the basic educational skills in relation to mathematics such as skip counting, it also familiarises them with concepts such as addition, subtraction, forward and backward counting. When children learn about the present, past and future, which basically refers to their personal experiences, they begin to put their lives in logical order and to make sense of what happens around them. As **Albert Einstein** once said, **"The reason time exists is so that everything does not happen at once"** Knowing that time occurs in a chronological arrangement is critical to a child's construction of time, in which the cultural aspect of time is being addressed by use of clocks and Calendars. The social aspect of time is also addressed by means of incorporating time into a routine, especially in **grade R** classes. Time is a modern human concept that needs to be learned.

Description of context of workshop

The lesson will start by means of a video, where participants will be exposed to the concept and the importance of time. They will be exposed to the kinds of materials that are needed as well as the instructions on how to involve learners in making their own clocks on paper plates, this will encourage learning and understanding by linking the teaching matter with their own hands-on-experiences. It is important that they are made aware of the dual purpose of numbers from 1-12 that appears on the analogue clock, which is the most confusing for learners when they learn time. The purpose of the number is always determined by the arm. They also need to learn to tell time on a digital clock which is always at their disposal in their homes. Learners will be exposed to completing worksheets that will emphasize mathematics vocabulary, they will also be made aware of the length of days and that time keeps repeating itself. They will be encouraged to show and tell the time on their devices by either counting forward or backward. Lastly, they will be involved in making their own wrist watches, play a game in which they will be collecting, analysing and interpreting time on a bar-graph using tablets.

Structure of the workshop

Activity	**Time allocated**
Watching a video	5 minutes
Feedback (what happened)	3 minutes
Creating own clocks	30 minutes
Manipulating the clock	12 minutes
Collecting data	5 minutes
Discussion and wrap up	5 minutes

Conclusion

Participants will be given an opportunity to engage with the presenter concerning the lesson. They will be expected to note the advantages and disadvantages of the method that was followed. The workshop will allow to explore a different approach to teaching time as it focused on making **"learning time"** easy and understandable to learners.

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Worksheets

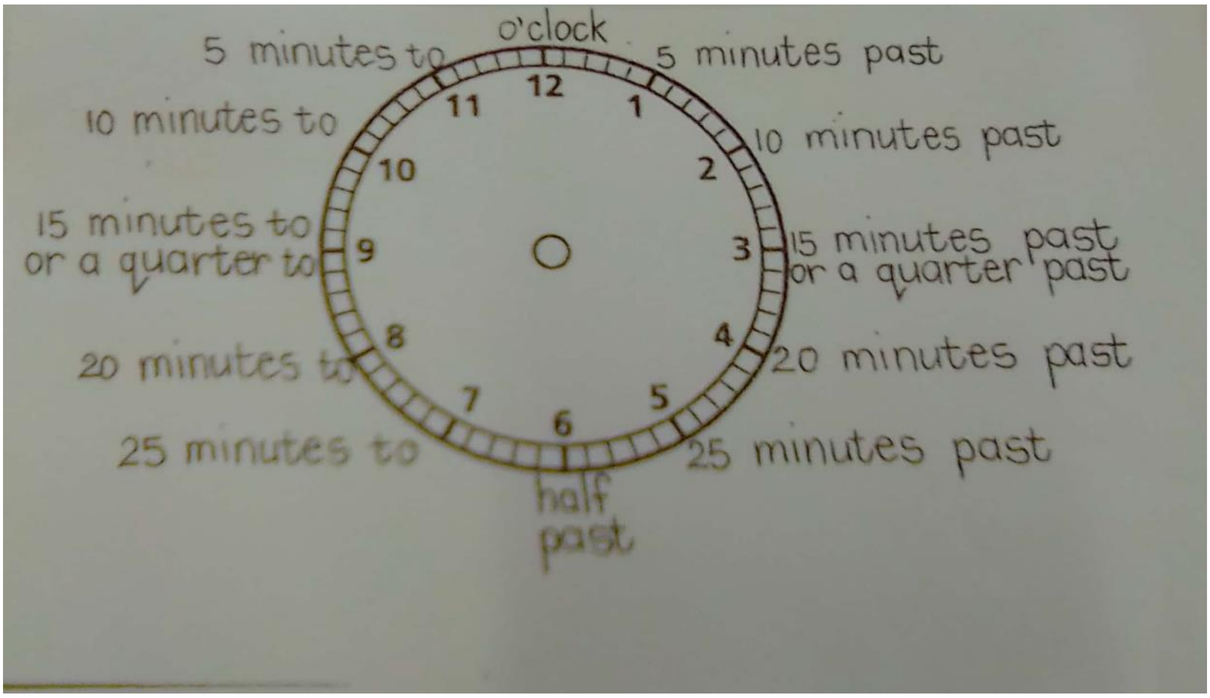
1. Instructions on how to make own clocks on paper plates
2. Counting forward and backward on an analogue clock
3. Telling the time on an analogue clock
4. What time does each of the clocks shoe
5. How does time pass, what is the time 1 hour later?
6. The digital clock
7. The right time
8. The bar-graph

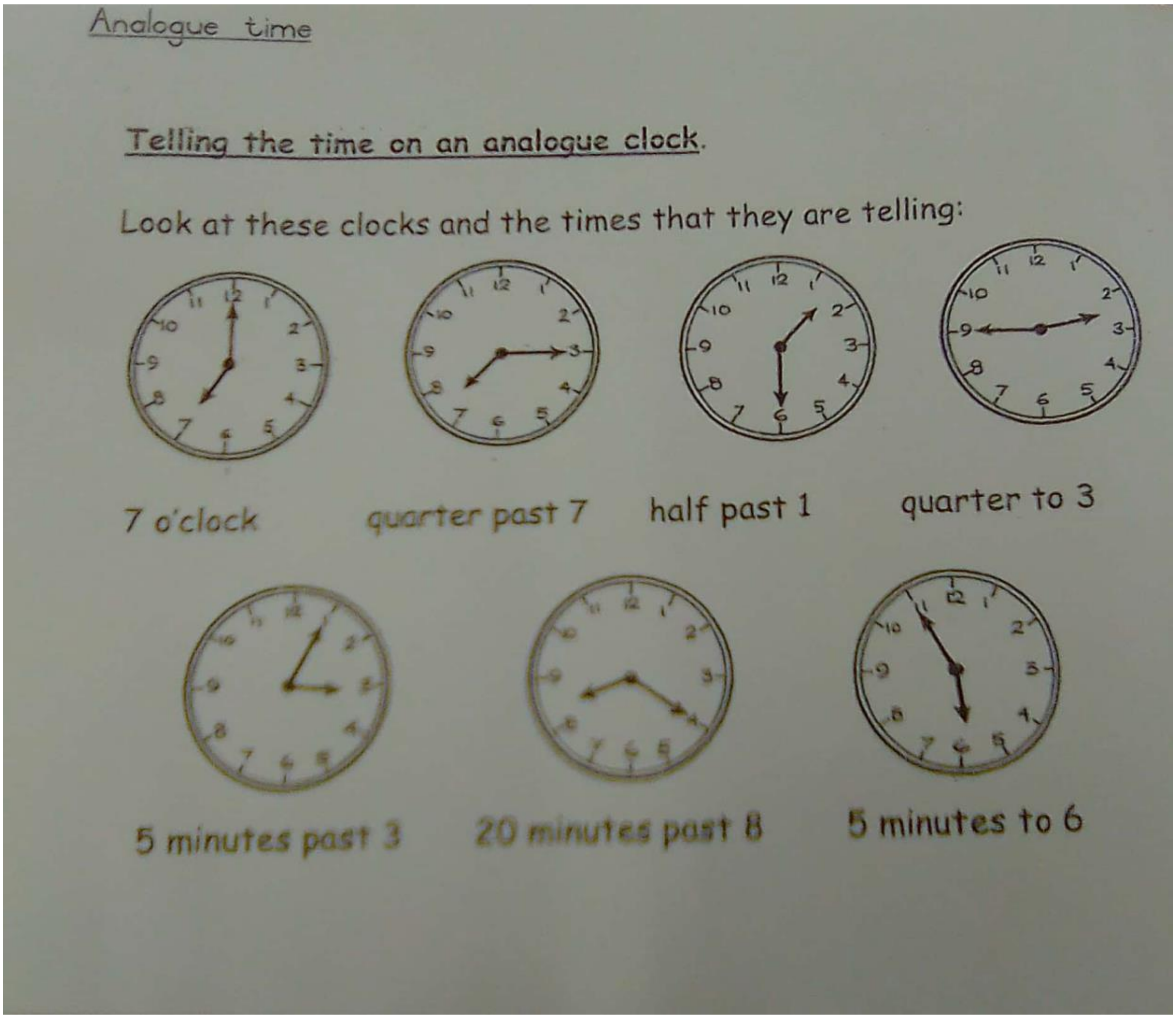
How to make a clock out of a paper plate

1. Cut out the hands from contrasting card, **making** one longer than the other for the minute hand.

2. Place the numbers round the **clock**, this is easier if you to the 12, 3, 6 and 9 first.

3. **Make** a small whole in the center of the **plate** for the hands and use the split pin to secure.





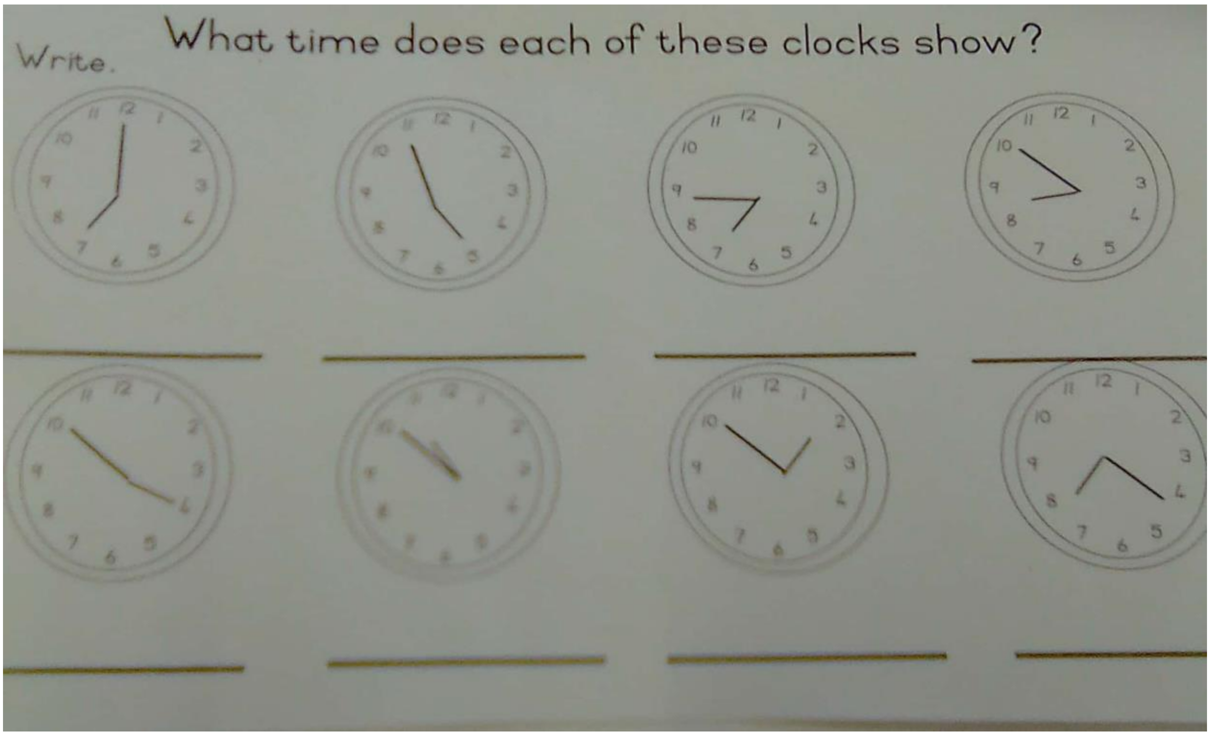
Analogue time

Telling the time on an analogue clock.

Look at these clocks and the times that they are telling:

7 o'clock quarter past 7 half past 1 quarter to 3

5 minutes past 3 20 minutes past 8 5 minutes to 6


Write.
What time does each of these clocks show?

How does time pass?

1 o'clock 2 o'clock 3 o'clock

= 2 hours passed

half past 8 half past 9 10 o'clock

= 1 and a half hours passed

What is the time 1 hour later? Write the answers in the blocks.

a. 12:30

b. 4:15

c. 8:00

d. 9:00

e. 9:30

f. 10:45

Answer these questions.

1. A movie started at 4:30 and lasted 30 minutes. At what time did it finish?
2. Nokuzula got into a taxi at 9:15. The journey took half an hour. At what time did she get off the taxi?
3. A roast needs to be in the oven for 30 minutes. If it is put in at 2:45, when must it be taken out of the oven?
4. Clara went into a shop at 6:00 a.m. and came out at 7:30 a.m. How long was she in the shop?

97

Complete the following facts about time.

1 day	=	____________	hours.
1 hour	=	____________	minutes.
1 minute	=	____________	seconds.

Fill in the missing times.
The first one has been done for you.

 = 7:50 p.m. = 19:50

 = ______ a.m. = ______

 = ______ p.m. = ______

 = ______ a.m. = ______

Work out the following times:

How long is it from 7:30 a.m. to 11:35 a.m.? .... hours .... minutes

How long is it from 8:45 a.m. to 12:15 a.m.? .... hours .... minutes

How long is it from 1:10 a.m. to 11:25 a.m.? .... hours .... minutes

Telling the time on a digital clock

am [ante meridiem, before midday]		pm [post meridiem, after midday]
1 o'clock	01:00 am	01:00 pm
2 o'clock	02:00 am	02:00 pm
3 o'clock	03:00 am	03:00 pm
4 o'clock	04:00 am	04:00 pm
5 o'clock	05:00 am	05:00 pm
6 o'clock	06:00 am	06:00 pm
7 o'clock	07:00 am	07:00 pm
8 o'clock	08:00 am	08:00 pm
9 o'clock	09:00 am	09:00 pm
10 o'clock	10:00 am	10:00 pm
11 o'clock	11:00 am	11:00 pm
12 o'clock	12:00 am	12:00 pm

We start at 3 o'clock, look at how we write minutes in digital time.

3 o'clock	03:00
5 past 3	03:05
10 past 3	03:10
quarter past 3 or 15 minutes past 3	03:15
20 past 2	03:20
25 past 3	03:25
half past 4	03:30

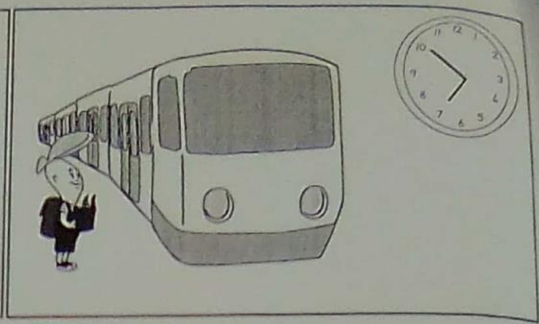
25 to 4	03:35
20 to 4	03:40
quarter to 4 or 15 minutes to 4	03:45
10 to 4	03:50
5 to 4	03:55

The right time

Write the time using a.m. or p.m.

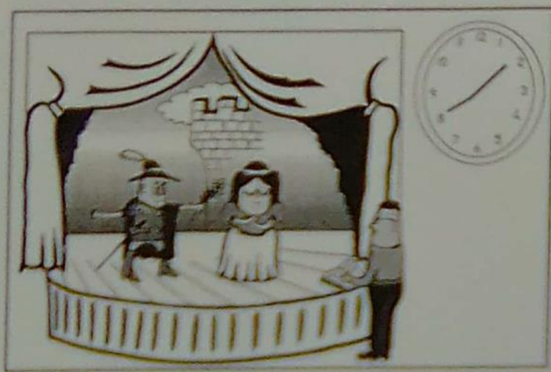
I walk the dog every night

at ______________________



The train leaves every morning

at ______________________



Tonight the concert starts

at ______________________

The plane will leave this afternoon

at ______________________

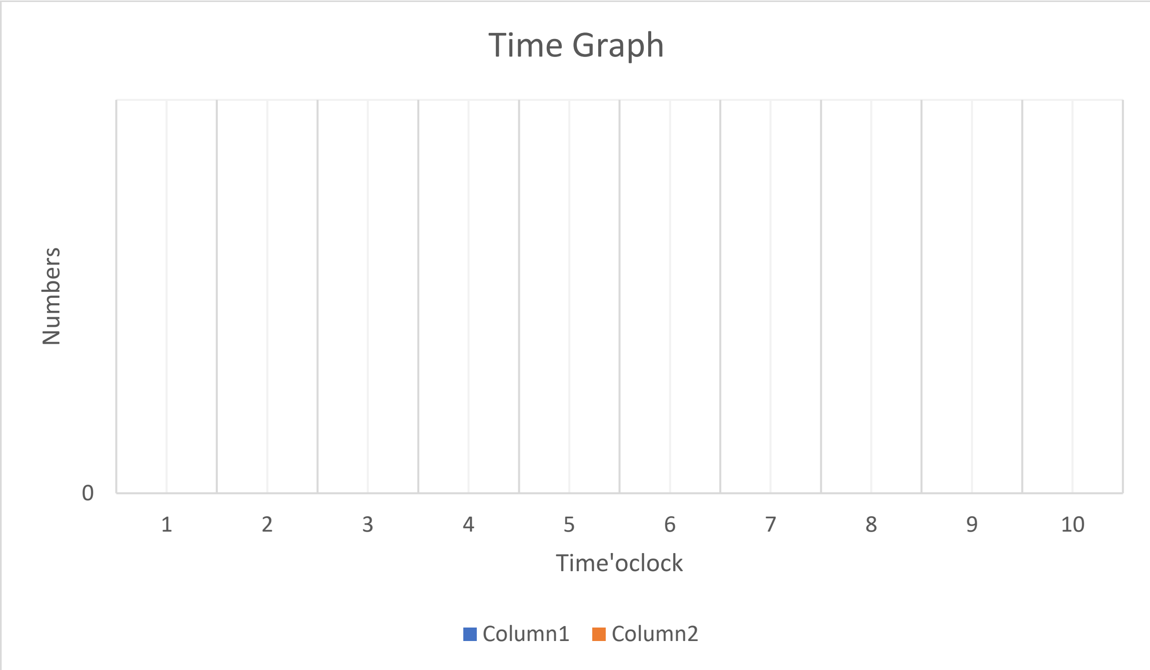
Pam missed the bus this morning.

It passed at ______________________

Tonight the game was over

at ______________________

Complete the graph by shading in the box each time you meet the particular time.



Implementing use of Technological Methods on how to teach rounding off and multiplication in Intermediate Phase

Rapulana Sanna Letshego

Thutotsebo Full-Service School

Target audience: Intermediate phase (Grade4-6)

Duration: 20minutes presentation plus 10 minutes discussion

Maximum number of participants: 25-30

Introduction

In a teaching and learning environment we have different kinds of learners who learn differently. According to four modalities of student learning that were described in 1992 study by Neil D, Fleming and Coleen E, Mills the acronym ‘’VARK’ stands for four different learning styles, namely : visual ,auditory, reading/writing and kinaesthetic, as teachers we have to cater for all leaners. We must use different methods to enhance teaching and learning. Learners must know that basics are very essential in mathematics. When rounding off and multiplying, learners develop critical and logical thinking. Teachers are life-long learners so they must always be eager to learn so as they transfer great and useful knowledge to learners, which will make learners great leaders of tomorrow.

Content

Mathematics is a subject which learner have found difficult and it lead to lack of interest. They feel that maths is difficult and that is not the case. How can we move from this notion of ‘’Maths is difficult?’’. Learners and teachers must know that maths is easy and fun, and technology makes maths easier. **COVID 19** has really made things challenging in education, learners don’t attend school well and they forget easily, so technology is very essential because it will allow them to learn through play. Media platforms like what’s-app, face- book, and also video games using cell phones or computers makes learners to have passion for maths, as they play they learn. Calculators are also essential as to check the answer after calculating each problem. Different teaching methods on how to multiply and round off must be shared amongst teachers, negatives and positives about methods addressed must also be shared.

Multiplication and rounding off are used on a daily basis, as you go for shopping it is important to estimate how much you will spend, for example when you go to Mr Price store to buy ten (10) jeans, you first look at the price of a jean, if a jean is R 100, you will have to multiply one hundred by 10 (R100× 10 = R1000) to get how much you will pay altogether. We also use our cell phone calculators to multiply and add as we busy shopping.

E.g. R100+ R100+ R100+ R100+R100+ R100+ R100+ R100+ R100+ R100= R100 × 10 = R 1 000 =

Therefore, learners can solve issues of their life's /world. You can multiply by estimating, estimate by rounding off.

Conclusion

Teachers must teach learners for knowledge not just for the sake of passing (quality vs quantity). As teachers we must equip ourselves with technology and pass it to learners, because technology makes maths easier. Technology can enhance relationships between teachers and learners, teaching and learning becomes more meaningful and fun.

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How I teach patterns in the Foundation Phase

D S Makgele

Relebeletse P School

Target Audience: Foundation Phase

Duration: 1 hour

Number of participants: 100

Teachers and parents are concerned about the covid- 19 epidemic which pushed to investigate new technology and their efficacy. It is worth noting that technology is evolving at a high speed these days. Currently and in the future, technology will play more important role in children's education, which in a process poses a variety of problems for parents and teachers due to non-exposure. Teachers need to keep up with technological innovations to improve their knowledge and that of their learners (Charlesworth, 2014). Hence it is critical to use technology in the classroom.

INTRODUCTION

Learners like looking for patterns in their surroundings. These patterns assist learners in comprehending change and the fact that things change through time. Learner's prediction, logical and reasoning skills are developed when learning about patterns. It is critical for teachers to understand the developmental sequence for teaching patterns as well as the integration of patterns in other subjects. Because learners learn about patterns in the early years of school, it is critical to follow the right developmental order while teaching patterns.

CONTENT

Firstly, I will take my audience through the developmental process of teaching patterning skills and show them how they may teach their learners the same developmental patterning skills.

Recognise a pattern

↓

Describe a pattern

↓

Copy a pattern

↓

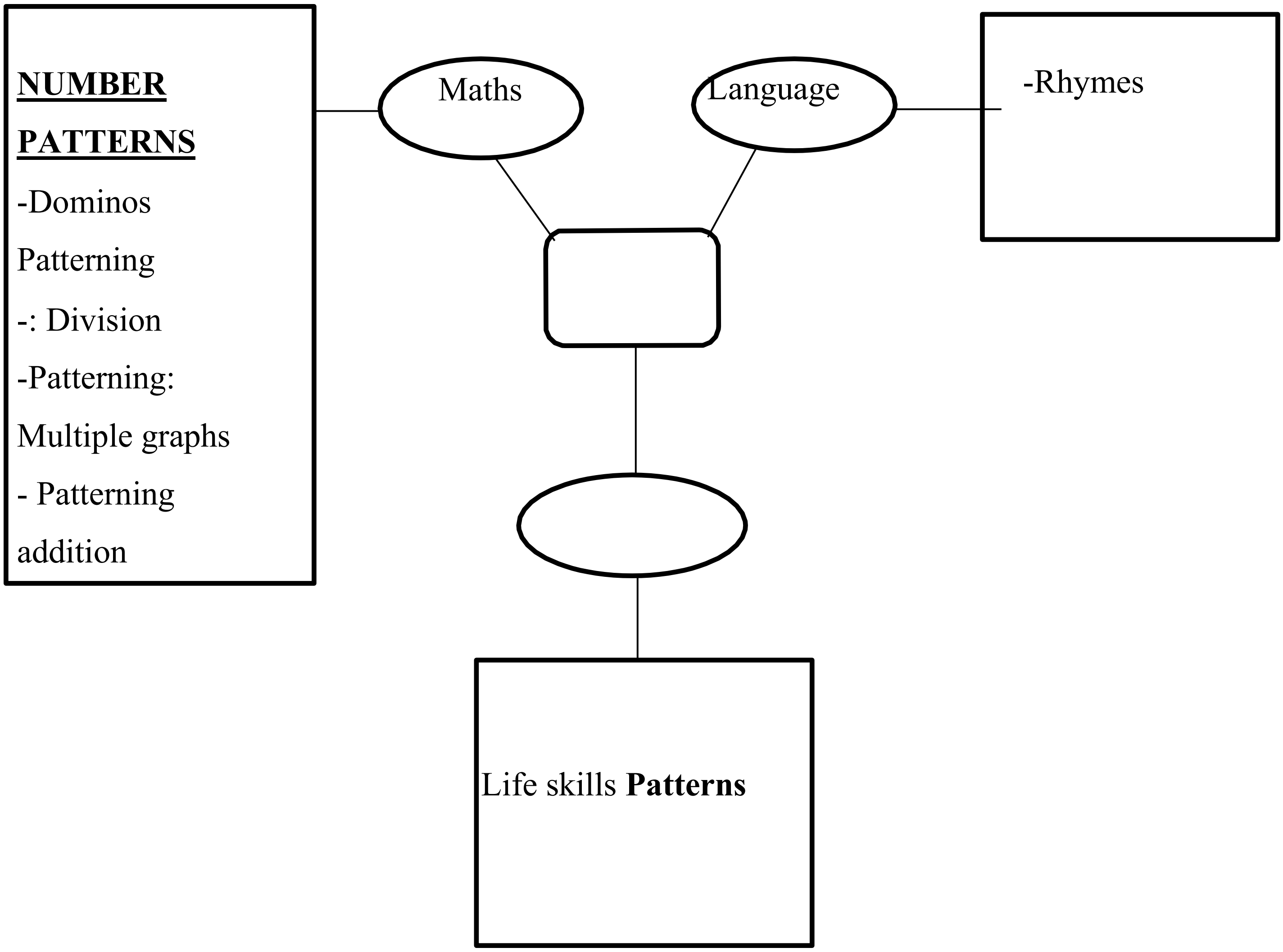
Extend a pattern

↓

Create own pattern

When I create a lesson plan, I consider each learner's unique learning styles, which might include modalities like auditory, visual, kinaesthetic, and multimodal preferences. I will show how I use computer and calculator exercises to incorporate diverse patterns in some subjects that suit to each learner learning style. I will describe the patterns stated on the mind-map and share the approach.

GROWING PATTERNS



KINAESTHETIC PATTERNS

-Body movement activities

SYMMETRIC PATTERNS

Pictures/shapes

MY EXPERINCE

I realized that teaching through manipulatives makes complicated thoughts and concepts easier to understand. The learners are drawn to the objects, and as a result, they pay close attention to the instruction. I constantly encourage my learners to explore the objects that will be used so they will not forget since the objects will the first thing that comes in mind , and the will be able to recollect every step/everything they performed with it. The use of manipulatives made teaching easier and interesting.

Conclusion

Teaching and learning of patterns equip learners with skills and knowledge that will be useful in their lives. Patterns are used by many professions to anticipate, estimate, and solve problems. When geologists seek to predict the earthquakes and volcano eruptions, for example, they can look for patterns in past seismograph data, atmospheric data, or even animal behaviour. Bankers also use past stock price and interest rate data to forecast how financial markets will evolve in the future. When tilling the floor, the builders employ patterns as well. It is worth noting that math concepts such as functions are built on the foundation of patterns.

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The use of technology in the teaching and learning of mathematics

Shamain Kamele Kamohelo

Primary School Bloemfontein

Target audience: Foundation Phase Grade R-3

Duration: 20 minutes Presentation plus 10 minutes discussion

Maximum number of participants: 100

INTRODUCTION

According to many educational researchers, some young children arrive at school with technology knowledge. Even pre- scholars might have had experience with technology such as Xbox, Wii Nintendo, video games. Interactive websites and electronic media system such as leapfrog even cellular phones. Where possible foundation phase teachers have to be acquainted with the popular technology and develop a plan for incorporating technology in their classrooms. Children can enjoy variety of educational software. The internet provides many learning opportunities.

CONTENT

Technology provides additional opportunities for learners to see and interact with mathematical concepts. Learners can explore and make discoveries with games simulations and digital tools. One excellent platform for teachers and learners is the web-based graphic calculator.

Although computers can never replace the opportunity to explore mathematics on concrete level, it can help Children bridge the gap from concrete to abstract. Children can learn mathematical concepts from software that presents a task, ask response and provides feedback.

Technology is the convenient way of teaching mathematics hence learners spent most of their time at home due to Covid-19, Zoom, whatsapp, Facebook,google meet ect, can create learning platforms for learners during this new normal.

CRITICAL QUESTIONS

People also ask:

1. How will technology be more effective in teaching mathematics?

2. How is mathematics used in technology?

3. What is the use of technology in the teaching and learning?
4. What is the relationship between mathematics and technology?
5. How should we use technology in teaching mathematics so that children can learn, given what we know about learners and how they learn?
6. How do we know whether what we are doing is working? If not; What must change?

Conclusion

First of all schools should be made user-friendly to technology for mathematics, e,g WIFI and all necessary electronics should be installed, through which internet can also be accessed. Electronic appliances such as PC computers, Laptops and tablets should be availed at schools for technology in mathematics. Schools starting with Foundation phase as priority should be supported with all aspects needed in technology for mathematics including books. Learners should be encouraged to use cell phones from their respective homes. Introduction of technology in Mathematics, at earliest age will make it easy for learners to successfully move from one level to another with good understanding of mathematics as a subject.

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