

Problems with Word Problems in Mathematics

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INTRODUCTION

Many teachers complain that learners find word problems in mathematics more difficult than straight computation, and that many learners dislike and even fear word problems. Different reasons are given for this phenomenon, of which the most common is that learners cannot read with understanding and therefore do not know what is required of them. One way to resolve this dilemma is simply to sidestep it – teach the methods, tools, techniques, and formulas which society believes make up mathematical knowledge and then test and examine in such a way that this kind of knowledge will ensure a good pass rate. Although this was a popular and common choice in the past, extensive research and practical (and personal) experience show clearly that this type of mathematics education is no real education at all and does not equip learners to deal with mathematical problems or to cope with further education in mathematics. For learners of all ages to become successful users of mathematical tools and mathematical thinking and problem solving strategies, they need to learn their mathematics with understanding, be able to relate to it, and try to make personal and collective sense of what they are learning. This can only happen when the mathematical ideas which we want to develop are embedded in situations that provide the possibilities of making connections to previous experiences, knowledge, and needs. For this, we as educators need word problems, i.e. situations which can give reasons for and meaning to the mathematics we want learners to acquire. However, although we therefore need word problems, the reasons why learners find word problems difficult are extremely complex. One of these reasons may be called the “straightforward” language problem – the reader’s lack of knowledge of the words used or the syntax where second-language or third-language speakers are obviously at a great disadvantage. However, in this paper I explore some of the other reasons why word problems are found to be difficult and unpleasant. I believe that these other reasons also have deep ties with literacy, because they involve perceptions and beliefs held by the learner about mathematics, about what is expected of her, and about her teacher, as well as the teacher’s beliefs about mathematics, how mathematics is learnt, and about appropriate learner behaviour.



WORD PROBLEMS – A COMPLEX ISSUE

It sounds reasonable to assume that, when learning mathematics, the learner’s ability to understand the language of instruction and also her level of reading comprehension play an important part in successful learning. We can have no quarrel with this, but there is a danger that functional literacy (including reading comprehension) can be interpreted too superficially, without taking into account all the many factors that may prevent the learner from making sense of what she is reading. Take for example the findings of the Third International Mathematics and Science Study–Repeat (TIMSS–R) (Howie, 2001) which show that, in South Africa, learners who seldom or never spoke the language of the test at home achieved lower scores than those who did. Yet in other countries, for example Malaysia and Morocco, the opposite was true – an observation that runs contrary to common sense expectations.

¹ Hanlie Murray passed away in January 2012. This previously unpublished article (prepared for a symposium at the 2001 International Literacy Conference in Cape Town) is published here posthumously as a tribute to her memory.

To highlight and illustrate how very complex a seemingly straightforward task of reading a piece of text really is, I would like to use an example from primary school mathematics. Let me state immediately that we shall not be looking at text which describes a mathematical concept or process, but at simple descriptions of “real life” situations as posed to young children in the form of word problems for them to solve. No mathematical terminology except number names and number symbols are used. Such contextualised problems, sometimes called story sums, should form a major part of the young child’s mathematical learning programme. When children are encouraged to solve word problems, their informal knowledge is elicited, they can relate to and make sense of their school mathematics, and they gain experience with a variety of mathematical structures and processes that can be developed and refined (Murray, Olivier & Human, 1998). Such word problems should therefore necessarily be simple and involve everyday situations and objects that young children can identify with. Therefore, when a teacher poses a word problem, either verbally or in writing, and the learners do not respond, she may say that they do not (or cannot) listen or that they do not (or cannot) read or do not read with sufficient care. Is this really the reason?

WORD PROBLEMS – BARRIERS TO UNDERSTANDING

To ensure a common understanding, the examples of problems will be limited to early mathematics. For the purposes of this paper, we shall take a word problem as a problem with a story or (pseudo) real life situation as context, and not simply a calculation. For a child to understand and respond to the problem posed, the language and grammatical constructions used when the word problem is formulated are obviously crucially important. This is self-evident. In addition, the following factors have been proved to be *as important* (compare Fischbein, Deri, Nello & Marino, 1985, p. 5).

1. The mathematical structure of the problem
2. The number sizes and kinds of numbers involved
3. The context used for the problem, for example, shopping, sports, a trip
4. The learner’s beliefs about what mathematics is and what the teacher expects from her
5. The teacher’s beliefs about what mathematics is, how mathematics is learnt, and how children learn mathematics.

The above factors can be divided into two very broad groups. The first group involves subject-related factors and the other group the socio-psychological factors. However, we need to keep in mind that the two groups cannot really be separated, as learners’ and teachers’ beliefs and attitudes influence not only the quality of the mathematics which is learnt, but also the *kind* of mathematics which is learnt.

1. The mathematical structure of the problem

There is an obvious difference between the mathematical structures of the following two problems:

Tsakane has R12. She spends R7. How much money does she have left?

Three friends must share 18 sweets equally. How many sweets must each friend get?

The first is a simple addition problem, and the second a simple (sharing-type) division problem. There are, however, at least 20 different addition and subtraction problem types with different mathematical structures, and at least three division types. This is the case for all the basic operations. For example, the adult may classify the following problems as straightforward subtraction situations similar to the Tsakane problem given earlier, yet they have quite different structures:

Tsakane has R5. She needs R12. How much money must she still get?

Tsakane has R12. Her brother has R5. How much more money does she have than her brother?

We have found that some of the different mathematical structures identified as suitable and necessary for early word problems are definitely more difficult for learners, but that it also depends very much on the amount of exposure learners have had to a particular structure. If they have not met it before, it may be extremely difficult. Many teachers believe division to be difficult and consequently hold back the division-type problems until later and by this very act prevent learners from becoming familiar with the mathematical structures of the different division problems.

2. The number sizes and kinds of numbers involved

Number sense does not develop uniformly for all numbers simultaneously; it rather develops for ever-increasing number *ranges*. A child may for example feel comfortable with numbers below 40, but be quite lost with larger numbers. When the child is then confronted with a problem involving numbers beyond her number range, she cannot generalise the knowledge she already possesses about the properties of operations and numbers to include the larger numbers as well. For example, she may know that $24 + 10$ is 34, but she does not generalise this to include $78 + 10 = 88$. Even worse, these unfamiliar numbers create “blank spots” in the word problem that may destroy the structure of the problem for the child, making it inaccessible. How many adults are not confused by the statement: “Tim earns two thirds as much as Andy”? Replace the two thirds by a whole number, and the problem becomes very easy: “Tim earns three times as much as Andy.”

Compare the following two problems:

Peter and Lise work in the garden. Peter works for 1 hour and Lise works for 3 hours. Their father gives them R20. How must they share the money in a fair way?

Peter and Lise work in the garden. Peter works for $1\frac{1}{2}$ (one and a half) hours and Lise works for $4\frac{1}{2}$ (four and a half) hours. Their father gives them R20. How must they share the money in a fair way?

The solution to the first problem almost leaps to the eye, but the second problem requires careful thinking and calculation, yet the mathematical structures are identical and they even have the same solutions!

3. The context used for the problem

So-called real-life contexts are not necessarily accessible to learners. Learners experience “real life” very differently from adults, and are familiar with very different aspects of real life. Our “real life” problems are in any case not really real, but rather “pseudo-real”, though even fairly realistic contexts may create barriers to learners’ sense-making. Here are four of the many ways in which the context can act as a barrier to understanding.

- ***Learners are not familiar with the context***

Urban children may not know how a farm functions, how cows are milked, orchards planted, fertilizer applied. When I had to prepare materials for a project in the rural areas of a northern part of South Africa I found that accessible contexts were extremely limited – learners had no familiarity with radio or television timetables, no transport except taxis, no supermarkets, few foodstuffs and sweets, limited sports, and almost no exposure to magazines and newspapers.

- ***The context has unpleasant connotations***

This includes unpleasant racial, sexual and socio-economic overtones, but personal and/or family-based problems can also be serious handicaps. During one of my classroom visits the teacher posed a problem involving a family trip which I thought was very suitable for the group of children involved. One little girl withdrew completely; the teacher then remembered that the girl’s father had abandoned the family the previous week.

- ***Limited contexts***

When the mathematical activities presented to learners use only a limited set of contexts, the concepts and skills developed by learners may not be transferable to other contexts. A good example of this is the number skills that many street vendors have developed in the context of money matters which they cannot apply or find difficult to apply to similar problems in other contexts.

- ***The problem has to be transformed or modelled by the learner before she can solve it***

In mathematics, the problem itself is very seldom solved, rather it is modelled through introducing numbers, symbols, diagrams, etc. and these models are then manipulated to find the answer. When the following problem is posed to a young child: *You have five sweets, and Dad gives you another two*

sweets. How many sweets do you have now? she may write $5 + 2 = 7$. This is a model of the problem. At an earlier level she may draw 5 sweets, another 2 sweets, and count them all. The drawing is also a model. But at the level before that, the child cannot replace the problem with a model. If she does not have the sweets physically available, she cannot solve the problem. She cannot use counters or drawings to represent the sweets; she needs the sweets themselves. The teacher may therefore pose a problem involving counters or crayons that the learners may solve with ease, yet the very next day the same problem, now involving biscuits or rabbits, may be completely inaccessible to some learners.

4. The learners' beliefs about what is expected of them

This includes beliefs about (i) the nature of mathematics and how mathematics is practised, (ii) how mathematics is learnt, and (iii) what the teacher values and how the teacher can be pleased. If the learner believes that mathematics is a 'box of tricks'; that the algorithms (methods) to solve a problem have to be retrieved from memory and applied, and that the teacher expects the learner to solve the problem in a particular way using a method preferred by the teacher, the learner is almost forced into perceiving the problem posed not as something to make sense of, and to solve as such. The learner actually solves a *different* problem: that of figuring out what the teacher would like her to do. The problem, and careful reading of the problem, therefore becomes less important than guessing what the teacher's expectations are (compare Schoenfeld, 1988, p. 85).

5. The teacher's beliefs about the nature of mathematics and how mathematics is best learnt

This will determine where in the learning sequence word problems should feature and how they should be used (for example as starting points to stimulate thinking or purely as applications). It will also determine how the problems are selected and sequenced, and how the activities involving word problems are structured – are the problems discussed, will different voices be heard, will learners be encouraged to reflect by inviting explanations and argument? All the above, in turn, influence learners' knowledge of the different problem structures and their willingness and ability to make sense of a given problem.

CONCLUSION

The word problems posed for early mathematical development do not use subject-specific terminology and are deliberately designed to offer situations that are accessible to young children, described in simple language. Yet many children have difficulty in understanding them. What can we learn from this?

I would like to suggest that functional literacy cannot be regarded as a general skill, but that it is domain-related, and deeply influenced by the socio-psychological factors that surround that *particular* domain for the individual reader or listener. The ability to read with understanding is not a "reading skill" as such, but depends as much on the reader's previous knowledge, her ability to make sense of what she is doing, and her beliefs about what is expected of her, including what she herself believes to be the *purpose* of the reading act.

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