The Effect of Contextualising a Problem on Learners' Approaches to Algebraic Equations

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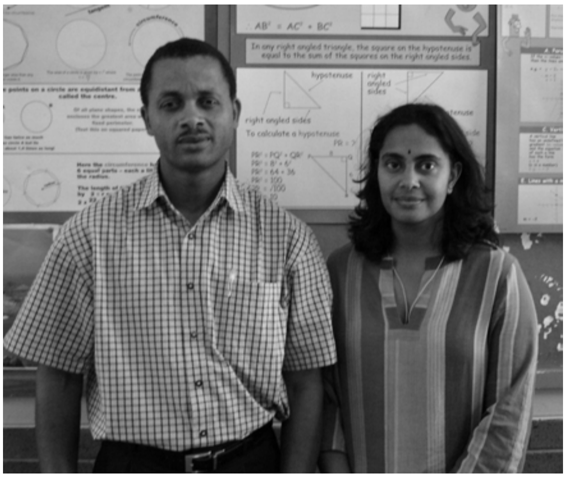

Introduction

Mathematical Literacy, offered as an alternative to Mathematics, has been introduced into the Further Education and Training (FET) phase in South Africa in 2006. The FET schools' Mathematical Literacy (ML) curriculum statement (Department of Education, 2003) stresses that the learning of content in the subject should occur within processes of solving real-life, contextualised problems.

The purpose of our small-scale study was to investigate the effects of using these kinds of contextualised problems with Grade 10 ML learners in the area of algebraic equations. Traditional methods of teaching mathematics, often involving textbook-based instructions have emphasized the use of procedures, rules and laws that learners had to reproduce in a test or examination. Many critiques of this kind of abstract, procedural learning have stressed the lack of meaning-making engaged in by learners (Schoenfeld, 1990). In our years of teaching experience, we have also often used these kinds of abstract, procedure-based methods when teaching mathematics. The first author is currently teaching ML in one of the three Grade 10 ML classes at his school, and wished to understand better some aspects of the impact of moving to more contextualised problem-solving approaches that have been effected by all of his department's ML teachers. Both authors attended the provincially organised training on ML in 2005. The first author, as part of an Honours project at the University of the Witwatersrand, decided to investigate some of the effects associated with shifting to the use of contextualised problems in ML.

Underlying our study was a concern with developing some of Kilpatrick *et al's* (2001) strands of mathematical proficiency, which include conceptual understanding, procedural fluency and productive disposition. Of relevance also to ML, De Lange (1996) views mathematics as a 'tool box' for solving problems. Our study was aimed at finding answers to the following questions:

(i) How are real-life contexts used in the learning of algebraic procedures such as algebraic equations?

(ii) What is the role of context in the learning of algebraic equations?

(iii) How does the learner feel it different when learning algebraic procedures in contextualised problems rather than in pure mathematical form?

The study was conducted within one Gr 10 ML class at the first author's school. The school is located in an inner city in Gauteng, and almost all of the learners in the school speak English as a second language. This factor raises a relevant further sub-question relating to the use of contextualised problem-solving based approaches, as a number of studies have documented concerns that the extensive presence of language in contextualised problems can act as an additional barrier to access and progress for these kinds of learners.

Process

In trying to find answers to the questions stated above we have chosen to use qualitative approaches to collect and analyse our data. A group of 27 learners at the first author's school was given a problem in two versions, one contextualised and the other non-contextualised, to solve. We used a fairly basic contextualised version of a problem. The aim of the problems was to uncover the effects of context in learners' approaches to problem-solving involving algebraic equations in mathematical literacy. The two versions were as follows:

Contextualised problem:

A hamburger costs twice as much as a cola. Themba buys four hamburgers and four colas for his friends. The total cost is R42. Find the cost of one hamburger and the cost of one cola *(from Understanding Mathematical Literacy Grade 10, 2005, Maskew Miller Longman).*

Non-contextualised problem:

Solve for x and y from the following equations:

$$4x + 4y = 42$$
$$y = 2x$$

The two problems above were accessible to the learners in terms of mathematical content and context, and the mathematical content of both problems was the same. Learners were also required to respond to five questions on their views about differences between the problems. A sample of four learners was drawn from the twenty-seven learners that participated in the study for further analysis in the group interview. Their selection was based on three aspects: the insights they provided in their written responses to the five questions on the role of context, use of a range of solution strategies within this sub-sample, and whether they were likely to respond to my interview questions. Learners' written work on the problems and their responses to the five questions suggested a break down into the following categories for analysis: solution strategies used, correct or incorrect solutions, comments about ease/difficulty of the two problems, and whether and how the problems were considered to be 'different'. A grounded approach (Glaser & Strauss, 1967) was therefore used in data analysis. Learners' responses using this categorization, are summarized in the table.

Contextualised Problem		
Category		**Freq**
Solution strategy	Trial and error	11
	Division by 8	7
	Division by 4	4
	Division by 2	4
	Use of algebraic procedures	1
Correct/ incorrect solutions	Correct	5
	Incorrect	22
Difficult/easy	Difficult	2
	Easy	16

Non-contextualised Problem		
Category		**Freq**
Solution strategy	Correct use of algebraic procedures	1
	Incorrect algebraic manipulation	26
Correct/ incorrect solutions	Correct solutions	1
	Incorrect solutions	26
Difficult/ easy	Difficult	24
	Easy	1

11 learners considered the problems to be different from each other, whilst 10 learners felt that they were the same, with 'sameness' often related to both questions involving R42. Of the 18 learners who commented on the problems, 16 stated that the second non-contextualised problem was considered to belong to more formal kinds of algebra – 'equations' or 'solving *x* and *y*'. Backing this sentiment up, 2 learners stated that the second problem, unlike the first, was about '*doing mathematics'*.

Findings

Learners used a range of different strategies when solving the contextualised problem, with only 1 learner opting for a formal algebraic approach, reflecting Boaler's (1999) finding that traditional school mathematics is poorly remembered and rarely transferred into realistic problem-solving. Fifteen learners preferred to use their own procedures such as dividing by eight, or by four or two because, as observed by Boaler (1999) learners do not usually see any connection between the mathematics they learned in school and the demands of their lives. In contrast, all the attempts at solving the second problem involved the use of algebraic manipulation, although only one learner out of the twenty-seven used correct algebraic procedures to solve the problem. Thus, contextualizing this problem appeared to open up avenues for learners to attempt solutions that did make sense – e.g. given the price of four hamburgers and four cokes, trying to work out the price of two or one of each by dividing by two or four respectively – even though errors in calculations made the overall solution incorrect.

Sixteen learners regarded the contextualised problem to be easy to solve in spite of the fact that the majority did not get a correct solution. Two learners said the problem was difficult - both of these were not fluent speakers of English which could have made the problem difficult to interpret. One learner who felt that the contextualised problem was easy, explained her reasons for this in the interview:

> *'I think it is because when we look at a story sum it gives a picture, it gives some kind of an insight. It was easy for you to think and answer at the same time, but problem two needs a lot of thinking.'*

Again, in contrast, 24 learners stated that the non-contextualised problem was 'difficult'. Out of the 24, 12 of them associated this commonly with figuring out *x*'s and *y*'s. One of the learners stated that *'problem two was difficult because you had to calculate it in many ways until you get your answer'*. In interviews, as stated earlier, learners confirmed that the non-contextualised problem was about mathematics and one of the learners stated '*I had no idea what to do'*. They also said it was difficult because it required them to *'apply what has been learned'* already and needed them to *'think a lot'* in order to solve the problem. This sense of the two versions relating to different 'worlds' in some senses, was captured within this written response to the question asking about whether the questions were 'different' from each other:

> *'In the first question, [it] had to do with money. And in the second question [it] had to do with solving for x and y. So the first one was a story sum and the second one was just maths. I preferred problem 1 because I understood the problem and what was asked for.'* (Brackets added in by us for clarity.)

The interview provided some more insight on what learners think about context and its role in developing learners' interest in engaging with mathematical problems. Contextualised problems were considered to be part of mathematical literacy, reflecting their classroom experiences. They confirmed this by stating that when they did mathematical literacy in their normal classes, they dealt with 'story sums'. They found it easier to engage in solving contextualised problems because they could 'interpret' them better than non-contextualised problems. Within our sample, there was a complete absence of comments relating to difficulties with negotiating the language within version 1 of the problem They further stated that contextualised problems did not require them to do a lot of calculations as pure mathematical problems did.

Conclusions

The findings in this study suggest that teaching mathematical literacy through the use of context may be helpful in improving learners' access to problem-solving, and in encouraging the idea that problem-solving involves making sense of a situation. In this approach, Kilpatrick *et al*'s notion of 'productive disposition' – developing a willingness to engage with problems and problem solving – was particularly highlighted. Our findings suggest that contextualising problems can work as a tool that allows learners to better engage with mathematical problems and mathematical ideas in a meaningful way. Learners' comments that ML learning in this school quite routinely seems to involve working with 'story sums' may point to one of the reasons why language has not appeared as a barrier to access in this study, as the first author points out that classwork in ML often involves whole class discussion of a problem context. Thus, negotiating language contexts in discussion has become one of the features of ML work in this school. As such, the kinds of contextual problems advocated for use in the Mathematical Literacy policy documents, may aid learners with more negative experiences of mathematical learning, to see mathematics as a powerful tool and way of thinking, that help them to work with and understand relevant problems.

A final point relates to the fact that the problem we chose to use, in its contextualised and non-contextualised versions, is familiar to both ML and mathematics. Thus, our finding of improved access to mathematical thinking may be worth considering within the context of mathematics teaching as well. Given the tradition of more non-contextualised, abstract work in mathematics lessons in comparison to the more routine use of contextualised problems in ML, it would be interesting to see whether similar sentiments are expressed by learners.

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