Some Pitfalls of Dynamic Geometry Software

Michael de Villiers
Dept. of Mathematics & Statistics, Kennesaw State University, USA

There is no question that computer technology has fundamentally changed the face of mathematics over the past 50 years so. As a kind of experimental laboratory, the computer has opened up vast new research fields like fractals and chaos theory, and rejuvenated many others. For example, even Euclidean geometry has received rekindled interest with the advent of dynamic geometry software packages like *Sketchpad*, *Cabri*, *Cinderella*, etc. The computer has also been used in proving some famous mathematical results like the Four Colour Conjecture by Appel & Haken in 1976. More recently, a mathematician called Wu has developed a computer approach for the automated proving of geometry theorems (see Elias, 2006).

In terms of teaching mathematics, the availability of different kinds of software seriously challenges the continued relevance of many aspects of what has always been seen as "traditional" mathematics. Is it really still relevant to spend hundreds of hours drilling and exercising learners or students in, for example, factorising polynomial expressions, particularly complex ones, when there is software readily available that can do it far more quickly and efficiently? The availability of graphing software (or just a spreadsheet) makes it possible for learners to explore problems of optimisation before (or even without) a course in calculus! Computer algebra software provides symbolic solutions to most algebra problems that students would encounter at the FET and undergraduate mathematics levels. Genuine real world data handling and analysis (as required in the new SA curriculum) is really only feasible with some form of computing technology, at the very least, a calculator with statistical functions. And so the list goes on.

Though several papers by others and myself have been written about the potential of dynamic geometry software, very little has been written about potential pitfalls. Though new technologies will inevitably make certain old skills obsolete, they will also require the development of new skills. For example, the replacement of the horse by the car as a major means of transportation required more people acquiring mechanical skills. Not only do new technologies require new skills, but also an awareness of new pitfalls, which may be created by them.

"No Change" Pitfall

One of the most obvious pitfalls that happens when new technology like dynamic geometry software is introduced in a school, is that rather than fundamentally changing their traditional style of teaching, many teachers just use it as a glorified blackboard. Sutherland (2005, p. 4) writes as follows in this regard:

"*When faced with a new technology we make sense of this in terms of our experiences of older technologies. So many teachers are likely to use digital whiteboards as an extension of the non-digital whiteboard. Many teachers are likely to use function graph-plotters as an extension of paper-and-pencil graph-plotting. Many teachers are likely to use dynamic geometry software as an extension of paper and pencil geometry. From this perspective we are likely to reject the new digital technology ... because used in this way the new digital technology is not as good as the old technology. In order to continue to use a new technology for doing mathematics we have to learn to use it in ways which transform mathematical activity, enabling us to do things which would not previously have been possible*."

It is, however, not only about changing teaching styles, but changes in emphases in the curriculum, new or alternative orderings of topics, etc. have to be carefully considered. For example, in a *Sketchpad* environment, one could easily take the built-in constructions and transformations as given "axioms" and start there, rather than first doing paper and pencil constructions like angle bisection, dropping perpendiculars, etc. and their proofs. One could then maybe later revisit these built-in *Sketchpad* constructions and transformations, and ask the question of how one would do them by compass and straight edge (and proving that they work).

"First Master Software" Pitfall

Another pitfall I've often observed by textbook authors and teachers, is the apparent assumption that children and students should first master the relevant software fully before they will be able to use it effectively in the classroom for teaching and learning. Nothing could be further from the truth! In order to drive a car effectively and safely, does one really need to know how an internal combustion engine works? Obviously the answer is, no. Similarly, children or students don't necessarily need to know the software inside out before they could effectively use the software to explore, learn, conceptualise, conjecture, etc. This can easily be achieved by providing students with more or less ready-made sketches that only require dragging, and perhaps clicking animation or construction buttons.

Another possibility is to only develop or expose students to the specific software skills necessary for a particular learning context. For example, at our Westville Campus a few years ago, I acquainted all our Primary Mathematics Education students with just the skills from the Transform and Construct menus to use *Sketchpad* effectively for an exploration of the basic transformations and some tessellations. (See photograph showing some of these students working with *Sketchpad*).

"Construct Dynamic Figures" Pitfall

Coupled with the preceding pitfall, and perhaps influenced by a misguided interpretation of constructivism, it is also quite common to find textbooks and teachers first requiring learners or students constructing dynamic geometry figures like squares, rectangles, etc. before they are allowed to explore their properties. This is really putting the cart before the horse! The problem with this is threefold:

Firstly, it requires a good knowledge and expertise with *Sketchpad*, *Cabri*, etc.

Secondly, this activity may take so long that the learners never get to the point of what they are supposed to learn or do (e.g. exploring properties of a figure or making a conjecture).

But thirdly, and more fundamentally, it completely disregards the fact that conceptually the activity to construct, for example, a dynamic square is at a much higher cognitive level than that of exploring and examining its properties! In order to construct any dynamic geometric configuration, requires a solid understanding of necessary and sufficient conditions, which according to the Van Hiele theory first occurs on Level 3, while that of visualising and exploring the geometric properties of objects are at Levels 1 and 2. So the whole irony of the situation is that learners are expected to first operate at Level 3 so that they can learn properties and concepts at the lower levels!

Asking learners or students to construct their own dynamic figures is of course a very good learning strategy, but this can only meaningfully occur after students have already learnt and understood all the properties of the figures concerned.

"Painless Learning" Pitfall

Another pitfall is to imagine that simply presenting or allowing an investigation of a problem or a theorem by means of dynamic geometry automatically makes geometry learning "easier" and "painless". Like any technology, dynamic geometry cannot offer a magical panacea for learning geometry by a process of automatic osmosis simply by staring at the beautiful, moving pictures on the screen. Unless the learner or student critically engages or is carefully guided to observe and examine what is happening on the screen, very little learning may actually be taking place.

A personal case in point was with my 4^{th} year pre-service teachers in 2005 while teaching a course in geometry using *Sketchpad*. One of the students wrote praisefully in her reflection how using the software had so wonderfully helped her to now " fully understand the theorems and proofs" so much better. However, when it eventually came to the exams, I found to my shock that she had hardly learned any geometry! It seems that she had been merely impressed, and perhaps even "confused", by the colourful, dynamic displays! Despite (what I thought were) my best efforts to use *Sketchpad* mainly as a starting point to move on to proof and deductive reasoning, she'd not learnt to step back from the visualisation, and had not managed to move to a higher level of conceptual engagement.

"Visualisation always makes easier" Pitfall

Indeed, enhanced visualisation by means of software like *Sketchpad* may sometimes be both a blessing and a curse. In the hands of someone who is mathematically literate and can clearly distinguish between cause and effect, given and required, and so on, dynamic geometry is obviously a marvellous tool. However, for the mathematical novice it could perhaps sometimes be confusing or distracting.

For example, Lavy (in press) reports in a recent study that university students who were not given access to *Logo* performed better at a task (that involved making and developing a proof) than students who were asked to investigate the task using *Logo*. It appears that the computerised version did not facilitate the problem solving as expected, precisely because of the "*data and visual overload*" provided by the software. Indeed, the computer version seemed to make it more difficult for the students to identify the crucial variables, whereas in the non-computer version there were fewer distractions.

"Insufficient Rethinking & Evaluation" Pitfall

As already mentioned earlier, to start using dynamic geometry (or any other technology) effectively, one really has to radically and critically rethink the content, the aims, and the teaching approach one uses. Secondly, once this has been done, the implementation of a new approach has to be backed up by systematic evaluation and revision in a continual cycle.

For example, one of the main advantages of dynamic geometry is its accuracy, immediate visual feedback and the ability to check many cases in a short space of time. Therefore, learners and students are much more likely to be easily convinced about the truth of a result or statement, but that immediately raises the question of why do we then still need a deductive proof to make sure?

This is why I have frequently argued that it is far more meaningful to INTRODUCE proof within a dynamic geometry context, NOT as a way of making sure, but rather as a means of explanation, understanding, and discovery before dealing with the more formal and abstract functions of verification and systematisation (see De Villiers, 1997, 2003). Research by some of my post-graduate students such as Mudaly (2000, 2004) and Govender (2002) has, apart from giving valuable insight into learners' and students' thinking and needs, strongly indicated the viability of such an approach.

"Makes Practical Obsolete" Pitfall

Though new technologies, as mentioned earlier, make certain skills obsolete, this does NOT imply that all old skills and activities should now be made redundant. Yes, perhaps some skills and activities should be scaled down, but may still be just as important as before for conceptualisation purposes, or at the very least, to create some appreciation for the power of the computer.

For example, teachers may be quite easily seduced by the relative ease and efficiency by which one can create quite complex tessellations with *Sketchpad*, to completely leave out practical, concrete experiences for learners with different tiles. This is problematic, I believe, particularly for weaker learners, because they are more likely to need the tactile, kinaesthetic experience of physically handling and creating a tiling by hand. The physical processes of packing out, turning, flipping over, translating, fitting together, etc. are still fundamentally important. The same can be said for the importance of hands-on activities like paper folding, cutting out, patty paper explorations, etc., all of which form very important conceptual experiences which neither *Sketchpad* nor any other software can replicate.

The bottom line is that *Sketchpad* and other software was never intended to replace such important hands-on activities, but can be used in different ways to enhance and extend children's learning experiences. The experience of some point by point plotting for understanding how *Sketchpad* creates graphs is still important, but is tedious and inefficient if used for exploring the behaviour of graphs.

From my university days I recall how we carried out by hand the simplex algorithm for linear programming problems involving up to 10 variables, and solving 2^{nd} order differential equations with cunning substitutions and transformations. Though this sort of algebraic competence certainly needs to be scaled down with available computer software, some technical expertise by hand is still needed to assist conceptualisation.

"Proof as Verification" Pitfall

However, a major pedagogical problem and pitfall with dynamic geometry, I believe, remains to be to convince novice (and perhaps some more experienced) learners and students of a continued need for proof as the ultimate means of verification. Even with my 2005 4^{th} year university students (prospective mathematics teachers) who had successfully completed some advanced calculus courses and done an introductory course to the logic of proof, many still responded with exasperating comments like: "Why do we have to prove this result to make absolutely sure? I can see it is obviously true on the screen!", "We can see these lines are concurrent, why do we still logically have to verify that they are?", "*Sketchpad* shows that any quadrilateral tessellates, who do we have to prove it?", etc.

And some of these comments persisted even after we had done some *Sketchpad* activities from my *Rethinking Proof* book which are specifically designed so that students will make some false conclusions. The one activity involves a ratio of areas, which appears constant when using 2-decimal accuracy, but is not constant when the maximum 5-decimal accuracy is used. Since *Sketchpad*, like any computer programme cannot work with infinite decimals, this example is used to try and raise doubt about whether we can really be 100% sure that if some measurement or calculation remains constant, it is actually still constant up to the 100^{th}, 1000^{th} or millionth decimal.

Another activity involves the apparent concurrency of three lines, the non-currency of which only becomes suspect when dragging to really extreme cases, and only becomes clear-cut non-concurrent by enlargement with a large, scale factor (i.e. zooming in).

Despite my efforts to inculcate a more critical, formal attitude to geometry proof in these students, most of whom were going to become high school mathematics teachers, it seemed that I was unsuccessful with a small, but significant number of students. For example, despite emphasising that when proving geometry results we had to ensure that the proof sketch was sufficiently general, a few students in the exams, still drew an equilateral triangle or a square when asked to prove something in general for a triangle or a quadrilateral. This obviously led to false assumptions and invalid proofs, since they incorrectly assumed certain things as given from their special cases. For example, by drawing an equilateral triangle when asked to prove that the perpendicular bisectors of a triangle are concurrent, led them to assume that they always passed through the vertices, and so immediately invalidated their attempted proofs based on that!

Even more disconcerting about this particular case was that we had also in the class dynamically looked at the difference between perpendicular bisectors and angle bisectors by constructing them on the same triangle. Students then through dragging were led to notice, that in general, perpendicular bisectors and angle bisectors do NOT coincide, and that two of them would only coincide when the triangle is at least isosceles. Despite this visual experience and my cautions when writing up formal proofs away from the computer, a few of these students still chose special rather than general cases.

From a constructivist perspective of learning, however, this sort of thing is to be expected as students frequently understand and construct their knowledge in ways quite different from what is anticipated or planned, and confirms the basic thesis of constructivism that learning is idiosyncratic. Moreover, constructivist learning theory also tells that there is no such thing as a perfect teaching approach that guarantees perfect learning. Learners and students from different educational backgrounds and experiences are frequently going to make sense differently from the same activity. Some students may grasp an idea or set of related ideas very quickly and move on to a higher level, while others may require practical or visual experiences, etc.

While it may be quite easy to scoff at students' proof attempts and struggles to develop a more mature, rigorous view of geometry proof, let me finish with the following cautionary example from some of my own mathematical work recently. While working on an interesting result I'd discovered, I came up with the following useful Lemma and proof (which allowed me to prove the required result).

Lemma
The midpoints of the segments connecting the adjacent vertices of two parallelograms form another parallelogram (see Figure 2).

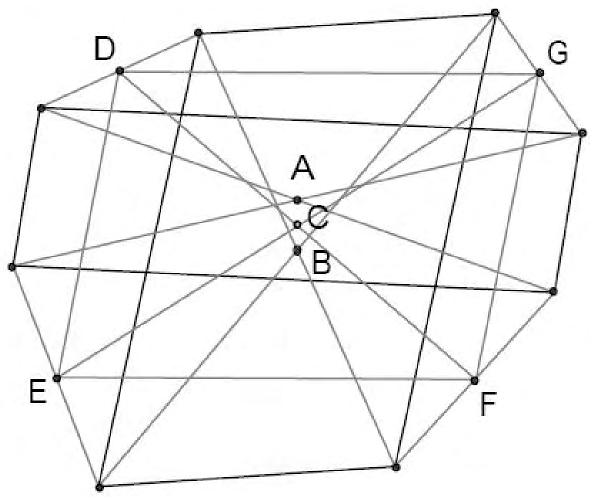

Figure 2

Proof
Assume equal point masses are placed at the vertices of the two parallelograms. Then the one parallelogram has centroid A and the other has centroid B, respectively at the intersections of their diagonals. Therefore, the centroid of all the masses must lie at the midpoint of AB, namely, C. Thus, A, B, and C are collinear.

But the midpoints of the segments connecting adjacent vertices, D, E, F and G are also the centroids of those pairs of adjacent vertices. Since C is the centroid of all the masses, it is also the centroid of D, E, F and G, and thus must also be the midpoint of both DF and EG. Consequently, $DEFG$ is a parallelogram.

A very, neat proof, isn't it? Simply using the idea of centroids it follows quickly and easily that *DEFG* is a parallelogram.

But is the proof VALID? Well, in fact, NO!

Only after critically reading a print out of my attempted proof of the Lemma above, did I realize that it was incorrect. I had made the classic student error of assuming *C* (as the centroid of *A* and *B*) lying at the intersection of the diagonals of *DEFG*, which is what I had to prove! The problem was that since I was using an accurate sketch within *Sketchpad*, *C* was placed there, but in a formal proof, I could not assume that. The only thing the last sentence in my "proof" really shows, is that *C* must lie half way between the midpoints of the diagonals, but does not at all prove that the two midpoints must coincide.

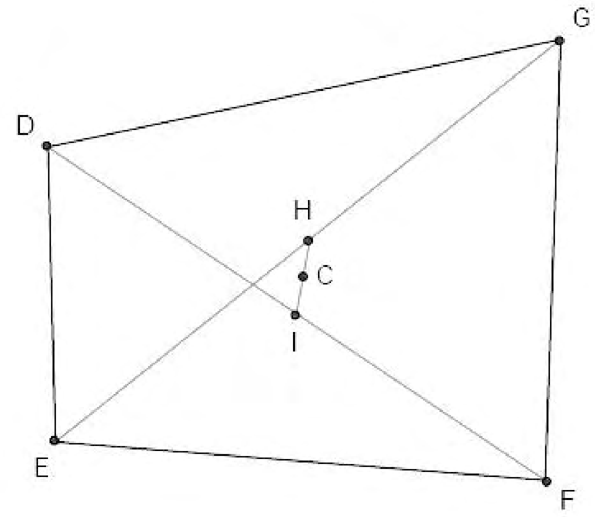

Figure 3

One could very well for the quadrilateral *DEFG* have the situation shown in Figure 3 where *C* lies halfway between the centroids *H* and *I*, respectively of the diagonals *DF* and *EG*. Only when *DEFG* is a parallelogram, will *H*, *I* and *C* coincide. But we cannot assume that *DEFG* is a parallelogram, because that is what we have to prove!

Though I've since developed a different, correct proof, and an article has been accepted for publication (see De Villiers, in press), this example shows how easily one can make a mistake. Now if we, as more experienced and mature mathematicians, can make such errors, how much more so are our novice learners and students going to make them?

More recently, on sabbatical at Kennesaw State University I've had the experience of some graduate students at the Master's level, attempt a proof of the concurrency of the perpendicular bisectors of a triangle by starting with a statement that the three perpendicular bisectors formed three pairs of congruent triangles (90°, *s*, *s*). Again this is assuming that they are concurrent, which is exactly what one is supposed to prove! If they don't meet in a point, then the three pairs of congruent triangles they are talking about won't be formed (in fact, six will be formed), so the whole rest of the attempted proof is false. Undoubtedly this error was stimulated by the fact that in *Sketchpad*, because it is accurate, the three perpendicular bisectors are shown to intersect in one point, and it is therefore quite understandable that some students would want to start there.

References

De Villiers, M. (1997). The Role of Proof in Investigative, Computer-based Geometry: Some personal reflections. Chapter in Schattschneider, D. & King, J. *Geometry Turned On!* Washington: MAA, pp. 15- 24.

De Villiers, M. (2003) *Rethinking Proof with Sketchpad 4*. Emeryville, CA: Key Curriculum Press.

De Villiers, M. (In press). A question of balance: an application of centroids, *Mathematical Gazette*, (Nov. 2007).

Elias, J. (2006). Automated Geometric Theorem Proving. *The Montana Mathematics Enthusiast*, Vol. 3, No. 1, pp. 3-50.

Govender, R. & De Villiers, M. (2002). Constructive evaluation of definitions in a Sketchpad context. Paper at AMESA 2002, Univ. of Natal, Durban.
(A copy of this paper can be downloaded from:
http://mysite.mweb.co.za/residents/profmd/rajen.pdf)
Lavy, I. (In press). Does visualization always help? The case of 'stars in cages'. *The International Journal for Mathematical Education in Science & Technology*.
Mudaly, V. & De Villiers, M. (2000). Learners's needs for conviction and explanation within the context of dynamic geometry. *Pythagoras*, 52 (August), 20-23.
Mudaly, V. & De Villiers, M. (2004). Mathematical modeling and proof. Paper at AMESA 2004, University of North-West, Potchefstroom.
(A copy of this paper can be downloaded from:
http://mysite.mweb.co.za/residents/profmd/vimmodel.pdf)
Sutherland, R. (2005). ICT and Learning Mathematics: Developing a Community of Practice, *Mathematics in School*, Sept., pp. 4-7.