The Grade 9 Maths ANA – What Can We See After Three Years?

Craig Pournara, Sihlobosenkosi Mpofu & Yvonne Sanders
School of Education, University of the Witwatersrand
craig.pournara@wits.ac.za sihlobosenkosi@gmail.com Yvonne.gubler@gmail.com

INTRODUCTION

The Annual National Assessments (ANA) were introduced to address South Africa's poor performance in international mathematics assessments such as TIMSS and SACMEQ. The Grade 9 test was first written in 2012 with a national average of 12.7%. In 2013 the average increased to 13.9% but then dropped to 10.8% in 2014. The overall picture is one of very poor performance with the result that many questions are being asked about the value and purpose of the ANA.

The Department of Basic Education (DBE) identifies three purposes for the ANAs: (1) a measure for the state to gauge improvement in the education system, based on learner performance year-on-year; (2) a diagnostic assessment to identify areas of weakness in learners' performance; and (3) to provide a model of good assessment practice. While these are all important goals, they are not necessarily achievable through a single instrument. However, this is not the focus of our article. Our purpose here is to report on a content analysis of the three ANA papers which we undertook in the first quarter of 2015, paying particular attention to cognitive demand and levels of difficulty. We share our findings on these aspects, as well as on trends that arise as we look across the first three years of the Grade 9 Maths ANA.

CLASSIFICATION OF ITEMS

Classification of items into taxonomies is never a simple matter, and there are no perfect taxonomies. We have attempted to use existing taxonomies with which teachers are familiar but we are not suggesting that these are definitive levels or categories.

LEVELS OF COGNITIVE DEMAND

Levels of cognitive demand describe the nature of the item in terms of the mathematical demand. An early Grade 9 ANA document (DBE, 2011) distinguishes three levels: knowledge of basic concepts, application of concepts and non-routine problem solving. However, we chose to work with the four levels of cognitive demand as described in the CAPS document since these are the levels that teachers work with regularly.

- *Knowledge* – requires straight recall, use of mathematical facts, identification and direct use of formulae.
- *Routine procedure* – well-rehearsed and familiar algorithms, simple calculations that may involve several steps, use of formulae requiring change of subject.
- *Complex procedure* – several calculations that are not all of the same type, complex calculations, route to solution is not entirely obvious, connecting representations, developing proof, requires conceptual understanding.
- *Problem solving* – unseen, non-routine problems, route to solution is not suggested, also requires conceptual understanding.

LEVELS OF DIFFICULTY

Levels of difficulty indicate how learners experience and cope with an item. This means we have to make assumptions about a 'typical learner'. In order to work with existing taxonomies it is necessary to define a typical learner to be one who entered Grade 9 with sound knowledge of Grade 8 mathematics, who is taught by a teacher who knows their mathematics and who teaches the whole curriculum to his or her learners in a school with the necessary resources for teaching and learning. We know well that this is not the reality for most Grade 9 learners in South Africa at present, and this is a challenge for the entire system. Nonetheless, following from our definition of a typical learner, we define the difficulty levels as follows:

- *Easy* – the majority of Grade 9 learners should be able to complete the question correctly, generally deals with one aspect only, is not complicated by non-essential features or seductive features that may lead to errors.
- *Moderate* – Grade 9 learners whose performance is 'average' should be able to complete the question correctly.
- *Difficult* – few Grade 9 learners are likely to get the question correct.

We illustrate the cognitive and difficulty levels with several examples in Figure 1.

Q9.1.3 (2014)

The sum of the exterior angles of any polygon is equal to __________

Knowledge – requires recall of facts.

Moderate – many learners will not recall this fact, more difficult than the sum of interior angles of a triangle or quadrilateral.

Q4.2 (2013)

Solve for x:

$$2(x-2)^2 = (2x-1)(x-3)$$

Routine – requires 4 well-rehearsed procedures.

Difficult – combination of several procedures, order of executing them matters.

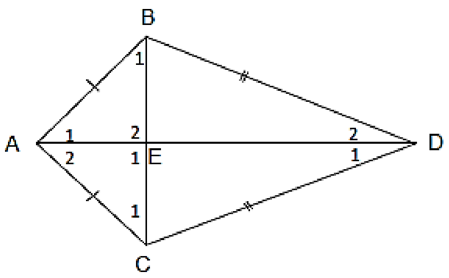
Q4.4 (2013)

Solve for x: $x^3 = 64$

Problem solving – not in Grade 9 curriculum but routine in Grade 10/11.

Moderate – trial and improvement with whole numbers will quickly reveal the answer.

Q6.3 (2012)



In the above figure AB = AC and BD = CD.

6.3.1 Prove that $\Delta ABD \equiv \Delta ACD$
6.3.2 Prove that $\Delta ABE \equiv \Delta ACE$
6.3.3 Prove that $\hat{E}_1 = \hat{E}_2$

6.3.1 *Routine, easy* – all information given.
6.3.2 *Routine, moderate* – need to deduce equal angles.
6.3.3 *Routine, difficult* – need to deduce from congruence and adjacent angles.

FIGURE 1: Examples illustrating levels of cognitive demand and difficulty.

Following Adler & Pournara (2013) and their teacher reference group, we work with a fourth category called *easy with a twist*. These are questions which would generally be classified as *easy* but which have been complicated in some way, for example through the use of particular numbers, inclusion of negatives, testing several concepts, or extreme cases. Such questions contain 'common traps' and thus may be more prone to error than questions at a *moderate* level since they may seduce learners into making errors. Some examples of questions with a twist are:

- Simplify: $\sqrt{16x^{16}}$ (Q1.1 – 2014) - The use of 16 as both coefficient and exponent is seductive and it is likely that learners will make an error with one of them, producing answers such as $4x^4$ or $8x^8$.
- Factorise: $9p^2 - 36q^2$ (Q2.4.2 – 2012) - Here we have a difference of two squares but the coefficients also have a common factor that is a perfect square. This question is potentially more error-prone than a similar case such as $9p^2 - 25q^2$ where both coefficients are perfect squares but with no common factor.
- While the example in Figure 2 might be considered a simple case of congruence, it is made more difficult by the overlapping triangles. The shape of the two triangles also suggests that they might be right-angled which means distractor B may be considered as the correct answer.

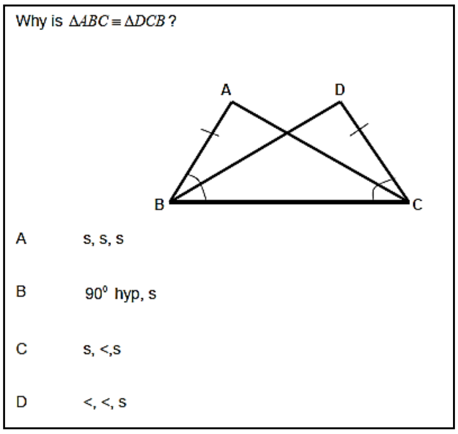


FIGURE 2: Question 1.9 (2012).

CLASSIFYING THE TEST ITEMS – WHAT WE DID AND HOW WE DID IT

We assigned each question to a particular Learning Outcome (LO) based on the CAPS document. In cases where an item dealt with more than one LO, we assigned it to the LO that was dominant. We also coded each sub-question on cognitive level and level of difficulty. This was done individually and then together as a team. Questions were only given one code for each of LO, cognitive demand and level of difficulty.

After the initial coding we undertook a systematic process of refining our coding. We describe this in some detail to substantiate our claim about consistency of our coding. We checked for consistency within a test, and then for consistency on similar items across tests. We also compared coding with the 2012 and 2013 coding (Adler & Pournara, 2013; Bowie, 2013). On several occasions this required us to re-check our interpretations of the codes. We also zoomed in on particular codes or combinations of codes (e.g. all the routine procedure, moderate items) and LOs checking for consistency in our coding across all three tests. We did the same with specific concepts and procedures, and this revealed interesting trends across the three years for particular content areas.

The advantage of engaging in this detailed coding process on all three tests simultaneously is that it enabled us to compare the tests in a more consistent way than simply comparing the coding of separate reports carried out at different times. This is important because the coding process is relatively subjective and so we often found ourselves changing codes and challenging our own interpretations of codes. We are confident that we have been consistent in our coding across all three tests but this does not guarantee that our coding is correct, nor that other coders would produce the same coding pattern.

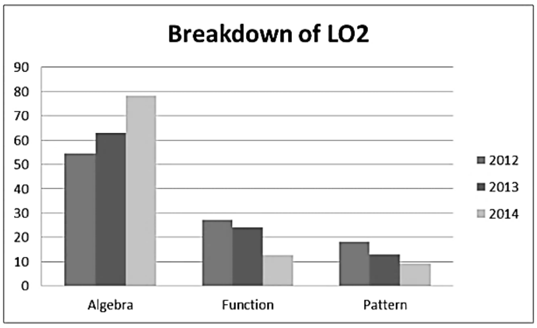
In the next section we provide summaries of our findings according to LO, cognitive levels and difficulty levels. In all three cases we compare our classification against the proposed weighting provided in the ANA document (DBE, 2011).

SPREAD PER LEARNING OUTCOME

The table alongside shows that the spread in 2012 and 2014 was a good fit with the proposed spread but this was not the case in 2013. In 2014, LO5 (statistics and probability) was not included. Although one might have expected the 10% of marks proposed for this LO to be evenly distributed across the other four LOs, this was not the case, with the bulk going to LO1.

LO	Proposed %	2012	2013	2014
LO1	15	13	9	22
LO2	35	39	38	39
LO3	30	28	19	30
LO4	10	10	16	9
LO5	10	10	18	0
TOTAL	**100**	**100**	**100**	**100**

Given that LO2 attracts the highest number of marks, we investigated the breakdown between algebra, patterns and function within LO2. As can be seen in the graph alongside, algebra dominates the LO2 items and this has gradually increased over the three years with the consequence that the other two sections have been allocated fewer marks each year.



COGNITIVE LEVELS

The table below shows that our overall classification of cognitive levels was quite different from that of the test developers. We classified fewer items as being *knowledge* and also fewer as being *problem solving* in each year. Inevitably we then had more items as *procedure* (routine or complex). Combining these codes we have classified the 2013 paper to be of lower cognitive demand than the 2012 and 2014 papers, with 85% of marks classified as routine or complex procedure and only 4% as problem solving - well below the proposed weighting.

Cognitive level	Proposed %	2012	2013	2014
Knowledge	25	19	11	13
Routine procedure	60	63	79	70
Complex procedure		11	6	10
Problem solving	15	7	4	7
TOTAL	**100**	**100**	**100**	**100**

DIFFICULTY LEVELS

Coding of difficulty levels is more subjective than coding of cognitive levels, and it is easy to slip between cognitive and difficulty levels. Interestingly, we classified many more items as *easy* than the developers had intended and consequently far fewer as *moderate*. The small percentage of *difficult* questions is worth noting in 2013. The summary shown below reinforces our earlier suggestion that the 2013 paper was easier than the other two years as shown by the higher percentage of *easy* items and lower percentage of *difficult* items.

Difficulty level	Proposed %	2012	2013	2014
Easy	25	44	49	39
Easy with a twist		10	9	12
Moderate	60	35	38	31
Difficult	15	11	5	18
TOTAL	**100**	**100**	**100**	**100**

COMPARING CONTENT AREAS ACROSS THREE YEARS

Based on our analyses, we identified several trends. In this section we discuss these trends with specific reference to items involving factorising, equations, geometry and problem solving.

Algebra

When we compare the items requiring factorisation across the three years we see a substantial shift in 2014, including three questions requiring the factorisation of a quadratic trinomial (see Figure 3). By contrast, there were no questions requiring such factorisation in 2013, although it is likely that many learners attempted to factorise $6a^3 - 12a^2 + 18a$ as the product of $6a$ and 2 linear factors (so this might be considered a question with a twist). It is also interesting to note that the factorising with grouping is only tested in 2012 and 2014. The more recent question is much easier, and we would argue more appropriate, than the 2012 question.

2012	2013	2014
Factorise fully: $8p^3 + 4p^2$ $9p^2 - 36q^2$ $tx - ty - 2x + 2y$ Solve: $x^2 - 2x = 0$	Factorise fully: $6a^3 - 12a^2 + 18a$ $7x^2 - 28$	Factorise fully: $3x^2y - 9xy^2 + 12x^3y^3$ $2(x + y) - t(x + y)$ $4x^2 - y^2$ $x^2 - 11x + 18$ Other questions requiring factorisation: Simplify: $\frac{x^2-4x}{x^2-2x-8}$ Solve: $x^2 - 5x - 6 = 0$

FIGURE 3: Items requiring factorisation.

The questions on equations also reveal interesting trends, as shown in Figure 4. Most noticeable is that there is no linear equation in 2014. We have included the last two questions from 2014 here because the instruction 'solve for x' suggests that procedures for solving equations should be used, although learners in Grade 9 do not have the necessary procedures to solve either these equations or the last question in 2013 and 2012.

2012	2013	2014
Solve for x: $3(x + 6) = 12$ $x^2 - 2x = 0$ $\frac{x+1}{3} - \frac{x-1}{6} = 1$ $2^{x+1} = 32$	Solve for x: $3x - 1 = 5$ $2(x - 2)^2 = (2x - 1)(x - 3)$ $\frac{2x-3}{2} + \frac{x+1}{3} = \frac{3x-1}{2}$ $x^3 = 64$	Solve for x: $(x - 2)^2 + 3x - 2 = (x + 3)^2$ $x^2 - 5x - 6 = 0$ $\frac{x+2}{3} - \frac{x-3}{4} = 0$ Solve for x without using a calculator. Show the calculation steps. $x = \left(\sqrt{8} + \sqrt{2}\right)^2$ $\sqrt{\frac{1}{\sqrt{x}}} = 3$

FIGURE 4: Items involving equations.

Geometry

Over the three years there has been increasing attention to formal proof in Euclidean geometry. We raise this as a concern because it appears to be beyond the scope of Grade 9. A word search in the CAPS document shows that the words *proof/prove/proving* occur only twice in total, and the word *rider* does not appear at all. Furthermore, the examples of 'riders' in the CAPS document (see pages 136-137) are all numeric. By contrast, the questions in the ANA require generalised formal reasoning, for example Question 6.3 from 2012 (see Figure 1) requires learners to prove two different cases of congruence for a general kite. Similar examples can be found in both the 2013 and 2014 papers.

Problem solving

There are two concerns about the problem solving questions. Firstly, we note, with Bowie (2013) and her teacher reference group, that there are insufficient questions on problem solving. Secondly, and more importantly, our concern is with the type of question that is considered as problem solving. In many cases these questions are simply content of the Grade 10 or 11 curricula, and would therefore be routine questions in those grades because learners would have been taught procedures for solving them. A selection of examples is given in Figure 5, taken from all three papers. As can be seen, the 2012 and 2013 questions would be routine for Grade 10 learners. The 2014 questions are more complex and although they are interesting questions, we suspect that they may not be accessible, even to the most talented Grade 9 learner.

<table>
<tr><td>Q2.5.4 (2012)
Solve: $2^{x+1} = 32$</td><td rowspan="3">Q14 (2013)
The 200 Grade 9 boys in a school play soccer, hockey or both. If 150 boys play soccer and 130 play hockey, calculate how many of them play BOTH soccer and hockey.</td></tr>
<tr><td>Q12.1 (2014)
Solve: $x = \left(\sqrt{8} + \sqrt{2}\right)^2$</td></tr>
<tr><td>Q12.2 (2014)
Solve: $\sqrt{\frac{1}{\sqrt{x}}} = 3$</td></tr>
</table>

FIGURE 5: Problem solving items from ANA tests.

The above examples do not have the characteristics of typical problem solving questions as one would find, for example, in the AMESA Maths Challenge or Maths Olympiad papers. In Figure 6 we provide four examples from Grade 8 and 9 Maths Olympiad first round papers. These examples illustrate a range of different questions involving number, pattern and logic, and geometry. They also show that problem solving questions are not necessarily all difficult – the first example can be approached by testing cases of prime numbers such as 2, 3, 5 or 2, 3, 7 etc. Olympiad tests typically have a multiple choice format (at least in the early rounds). This may need to be adapted for the ANA context.

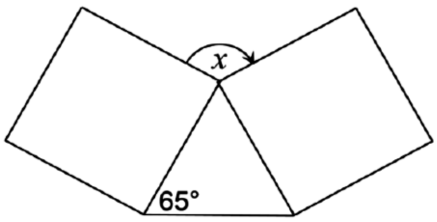
Grade 9 (2014) The product of three prime numbers is 42. The sum of these numbers is: (A) 12 (B) 13 (C) 14 (D) 15 (E) 16
Grade 8 (2015) If today is Thursday, what day of the week will it be in 150 days from now? (A) Sunday (B) Monday (C) Tuesday (D) Wednesday (E) Thursday
Grade 9 (2014) Two identical squares meet at a vertex as shown. The size of the angle marked x is:  (A) $105°$ (B) $120°$ (C) $130°$ (D) $150°$ (E) $160°$
Grade 9 (2013) A set of 12 numbers has an average of 18, but the smallest and largest have an average of 28. What is the average of the others? (A) 14 (B) 15 (C) 16 (D) 17 (E) 18

FIGURE 6: A selection of Olympiad questions.

CONTENT ON THE PERIPHERY

We distinguish between two kinds of content on the periphery: special/extreme cases and content that we describe as being 'in the shadows'.

The most obvious example of a special case is Question 8.1 (2012) which deals with properties of 3-D shapes. The chosen shape was a cylinder, which raises a mathematical debate about the number of vertices, faces and edges since many would argue that a cylinder has no edges or vertices. We note that there were no examples of extreme/special cases in 2013 or 2014.

There are some concepts which are in the curriculum but which lie on the periphery and are not central to Grade 9 mathematics. Most are given substantial attention in Grade 10. Typical examples are quadratic equations such as $x^2 - 2x = 0$ and $x^2 - 5x - 6 = 0$, as well as exponential equations such as $2^{x+1} = 32$ and $3^x = \frac{1}{9}$. While it is clear that both are part of the Grade 9 CAPS curriculum (see page 125) in our opinion it is not necessary to assess them in a Grade 9 national assessment.

CONCLUSION

In our content analysis of the 2012, 2013 and 2014 Grade 9 ANA tests, we have focused on concerns which arose for us. However, there are many appropriate questions and some questions that we particularly like, for example Q5 and Q8 (2013), and Q 11.3 (2014). Whilst the analysis suggests that the 2013 paper was easier than the other two years, one must be cautious in comparing across years because the tests are not identical, as illustrated by the questions on factorising and equations.

When we reflect on all that we have seen through our analysis, one of our biggest concerns is the level of demand of the geometry items. This comes at a time when Euclidean geometry has been reintroduced into the core of FET mathematics and so it is important that Euclidean geometry be taken seriously in the Senior Phase. However, this does require a premature rush to produce formal proof.

Finally, we believe that the notion of *question with a twist* is a useful idea for all teachers because it prompts us to be more aware of ways in which we may make a test question more prone to error without necessarily intending to do so. On the other hand, questions with a twist may be very productive for *learning* and so it may be wise to include questions with a twist in our selection of examples for classroom teaching because they will bring out the typical errors that learners make, errors which can then be addressed.

We close by offering the reader three examples of questions that we think have a twist, and we challenge readers to make minor changes to the questions to remove the twist and hence to reduce the likelihood that learners will be seduced into errors. We also challenge teachers to use such 'questions with a twist' in their teaching and to share their experiences with the LTM readership.

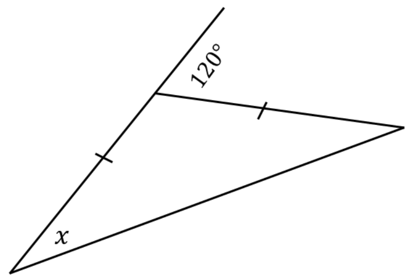
Solve for m: $m - 12 = 12 - m$	Simplify (no variable is zero): $\frac{2y}{x} \cdot \frac{xy^{-2}}{6y^2}$	Determine the size of x: 

FIGURE 7: Questions with a twist.

REFERENCES

Adler, J. & Pournara, C. (2013). *Report on the Annual National Assessment Grade 9 Mathematics Test, 2012*. Pretoria: Department of Basic Education.

Bowie, L. (2013). *Report on the Annual National Assessment Grade 9 Mathematics Test, 2013*. Pretoria: Department of Basic Education. Pretoria.

Department of Basic Education (DBE). (2011). *Annual National Assessments 2011: Framework for Mathematics – Grade 9.* Pretoria: Department of Basic Education.