Geometry from the World around Us

Dorit Patkin[1] & Ilana Levenberg[2]

[1]Kibbutzim College of Education, Tel-Aviv, Israel [2]Gordon College of Education, Haifa, Israel

[1]patkin@netvision.net.il [2]dveer@netvision.net.il

THEORETICAL BACKGROUND

Geometry forms an important component in both elementary and high school curricula. However, it is often perceived as being one of the most complex parts of the curriculum. Students frequently experience a sense of travelling to "an isolated island" where everything is structured in a "logical" or "unusual" way, without any relation to daily life.

There are various theories dealing with the development of pupils' geometric thinking. One of them is the Van Hiele theory. According to this theory, development in the study of geometry progresses in a hierarchical order through various levels of mastery, where partial mastery at a particular level is crucial but not necessarily sufficient for understanding on a higher level. Students cannot function on a particular level if they have not achieved mastery of previous levels. Therefore, those engaged in teaching geometry must relate to the various levels of thinking among students in the class, striving to overcome the students' difficulties in learning basic geometric concepts.

Crowley (1987) argues that the *type* of activities given to learners represents the most meaningful consideration in terms of the development of geometric thinking. His findings indicated that in order to enhance meaningful learning there must be complete compliance between the learners' level of understanding and the level of assignments with which they have to cope.

At the first stages, geometry studies involve *visualisation*. Pupils whose visual competence is not sufficiently developed will encounter problems in this subject. Moreover, according to Van Hiele's levels of thinking, visual competence constitutes an important factor when examining learners already at the first stage. Developing visual competence in the course of teaching aims to increase learners' mathematical power and promote their ability to solve mathematical problems. Furthermore, emphasizing the *aesthetic* aspect might improve learners' comprehension, increase their awareness of the importance attributed to examination and observation and, consequently, change their attitude towards the subject.

A recent study by Walker et al. (2011) investigated whether the development of visualisation skills in non-mathematical contexts may confer an advantage for geometric reasoning. The research findings clearly showed that the development of visual links has importance for, as well as very great impact on, the level of comprehension of geometrical content.

To sum up, various studies reiterate the significance of two key elements for the promotion of geometric thinking, namely *visualisation* and *application*.

THE ENVIRONMENT AS A SOURCE OF GEOMETRIC ACTIVITY

During geometry lessons, the use of all types of visual displays, pictures, presentations and movies, which show geometry in the pupils' environment (both natural as well as man-made), constitutes a bridge between the concrete and the abstract. These means capture the sense of sight, enhance awareness of aesthetics and help learners understand the functions of geometry in our life. In addition to encouraging and recommending the use of visual displays, it is important to emphasize the fact that teachers are not always required to prepare special aids. Every object around us can serve as an illustrative means for the subject. Even a football or an orange can be the beginning of a fascinating lesson. It all depends on the extent of imagination and creativity of teachers wanting to evoke interest and improve their pupils' understanding of geometry.

The significance of the types of activities presented in this article are corroborated by Hershkovitz, Peled and Littler (2009) by their highlighting of how important it is that pupils receive from their teachers open and varied assignments that promote creativity and incorporate the need for observation and the development of visual competence.

The purpose of this article is to share suggestions and examples of tried and tested activities designed to promote and develop geometric thinking. The activities are based on visual illustrations taken from the learners' environment. The suggested activities incorporate both natural and man-made examples which attempt to bridge the concrete and the abstract. These activities can serve as an introduction to a studied subject, the core of the studied subject or, alternately, they can be introduced in enrichment lessons or as part of a summary of a chapter.

The pedagogical and didactic functions of the activities are that they:

- Offer interesting and unusual mathematical activities to the pupils.
- Encourage mathematical engagement through experience and inquisitiveness.
- Develop the learners' ability to cope with problems taken from their daily environment.
- Present the relation between mathematics and other disciplines, such as biology, architecture, etc.
- Reduce anxiety of the subject.
- Create opportunities for mathematical activity for pupils who find the subject difficult.

PUPILS' EXPERIENCES WITH "GEOMETRY AROUND US"

Within the framework of enrichment advanced studies for gifted pupils with special interest in mathematics, we experimented with the learning materials presented in this article with 56 pupils in the 5th and 6th grades. The pupils experienced numerous activities during several lessons dedicated to various geometric topics (squares, circles etc.). A discourse about the question: "What have you learnt in geometry that you had not known previously?" was conducted after each activity.

One of the first activities introduced to the pupils was a photo of a garden shaped like an acute-angled triangle, the plant beds being concentrated at its vertex. The pupils were asked: "Where should we place a single sprinkler so that it irrigates all the plant beds?" The term "*in the middle"* came up in the pupils' answers; only a few of them, though, knew how to relate to the term "center of the circle". The activity also dealt with gardens shaped like obtuse-angled and right-angled triangles. Following this activity, terms like "center of a circle", "circumcircle" etc. evolved from being virtual or abstract concepts to real and applicable concepts.

Another activity centred on traffic signs and was performed with the pupils while touring a residential quarter and road junctions. We received the following responses from the pupils:

> *"I have not thought I could find mathematics on the street".*
>
> *"I have never seen any relation between things around my house and the teacher's explanations in class".*
>
> *"At first I did not understand what the teacher meant. How can something outside be connected to the drawing in the textbook?"*

After the various activities had been carried out, pupils were asked to develop by themselves similar activities associated with the various topics. At the end of the term we organised an exhibition, presenting photos taken by the pupils (some of them were even taken with their mobile phone camera when they came across a suitable environmental exhibit). We also displayed various questions formulated by the pupils based on the exhibit in the respective photos. Below are several examples of the activities carried out by the pupils.

EXAMPLES OF ACTIVITIES BASED ON NATURAL AND MAN-MADE ARTEFACTS

1. Vegetation in the desert – "Natural calipers"

The picture shows sandy soil and a desert plant, the ends of its leaves "drawing a circle" by means of the wind. The plant, of course, is the center of the circle and the size of the drawn circles depends on the length of the plant leaves as well as the force of the wind.

Suggested questions:

- Try describing in words the phenomenon presented to you.
- What can you say about the place where the grass grows in the drawn circle?
- Why do some plants draw small circles while others draw big ones?
- Are there other phenomena which form shapes similar to those presented in the picture?
- If you examine two plants which are close to each other, try drawing the circles created by the wind. Specify several options.

The above questions comply with the two first levels of the Van Hiele theory. In the first question, pupils have to identify shapes (1st level) while the other questions relate to the features of the shape (2nd level).

2. Butterflies

Photos of various butterflies were presented to the pupils. The activity facilitates reinforcement and development of the learners' visual skills, in addition to the inculcation of the concept of symmetry and its meaning in geometry.

Suggested questions:

- Try describing in words the exhibit in the photo.
- Make a note of what is similar and what is different in the butterflies in the photo.
- Try examining the term "symmetry" in relation to the different butterfly shapes.

3. Sites in residential areas

The immediate neighborhood of every pupil, or a picture or photograph from another country, can be a site which invokes mathematical activity. By way of example, the Bahá'i Gardens in Israel (above left) or the Alhambra Palace in Spain (above right) both constitute astonishing geometric shapes**.** Such pictures

can also lead to questions at the first and second level of Van Hiele's theory, namely questions relating to the identification of basic shapes and recognizing their features.

4. Man-made symmetry in places of worship

Buildings and places of worship, such as churches, cathedrals and monasteries, abound with interesting geometric patterns. Below is an activity that was carried out at the Carmelite monastery (in Haifa, Israel) which contains some wonderful floor designs.

Suggested activities for the pupils:

- When you enter the monastery please keep quiet and respect the place. Observe the uniquely-tiled flooring.
- How is this flooring different from previous ones which you have seen in the past?
- How many types of symmetry do you notice?
- Draw a straight line between the ends of adjacent leaves. Which polygon do you get?
- Repeat this with the inner leaves of identical colour. Which polygons do you get now? Are they regular polygons?
- Draw a straight line between the ends of every second leaf. Which polygon do you get now? Is it also a regular polygon? What is the relation between previous polygons and this one?
- Is it possible to get a regular polygon with an odd number of sides by drawing a line between the edges of the leaves?

5. Traffic signs – geometrical shapes from the street to the classroom

Pupils have to learn traffic signs as indicators of how to behave on the road. This can be seen as a combination of geometry studies and traffic signs as an international "language".

Suggested questions:

- Which geometric shapes familiar to you serve as traffic signs?
- What characterizes the triangle-shaped traffic signs?
- Why is the "Stop" sign different from other traffic signs? In what way is it different?
- Look at the "Do not enter" and "Yield" signs. Indicate special features about each of the signs.
- With reference to the "Do not enter" sign, estimate the ratio between the rectangular area of the sign and the area of the entire circle.

6. Car logos

The car industry incorporates many geometric shapes and features into their car logos.



Suggested creative activities for the pupils:

- Create a logo for a company by using three different geometric shapes. The logo can be designed only with a ruler and compass – as part of exercising the topic of geometric constructions.
- Choose a car logo and describe it in words by referring to its component shapes and elements of symmetry.

7. Urban centres

Urban centres with skyscrapers and differently shaped houses can serve for solid geometry activities.

Suggested activities for the pupils:

- Describe in words the skyscrapers in the picture.
- Try estimating their height. Explain how you did it.
- Try estimating the number of windows in the façade of the skyscraper on the right. Explain how you did it.

CONCLUDING COMMENTS

To sum up, geometry studies provide a rich environment for the purpose of developing mathematical thinking, developing logical thinking skills, using intuition and developing spatial orientation and acquaintance with the environment in our daily reality. Mathematical activities such as those suggested in this article enhance acquaintance with and inculcation of mathematical processes and the implementation thereof (Van Hiele, 1999), and improve verbal communication in general and mathematical communication in particular. It is hoped that the activities we have suggested in this article provide contexts for meaningful exploration and learning, while at the same time enhancing mathematical communication and engaging pupils with the beauty of geometrical design.

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