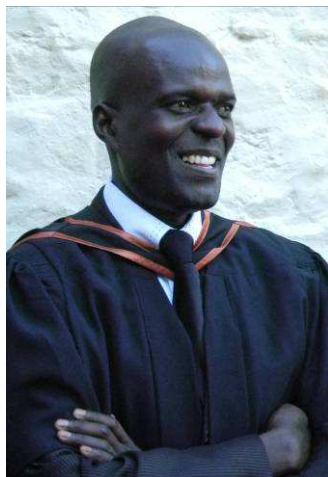


The Role of Physical Manipulatives in Teaching and Learning Measurement

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Measurement is a critical aspect of mathematics that affords opportunities for learning while applying and engaging with a host of other mathematical topics (Clements & Bright, 2003, p. xi). Although measurement is a theme that permeates all areas of mathematics as well as day-to-day life, research has shown that many learners find it an aspect of mathematics that is difficult to grasp, with learners often “not understand[ing] the attribute being measured or the units that are used for measurement” (O’Keefe & Bobis, 2008, p. 391). Learners often find particular difficulty in determining the surface area and/or volume of a given object. Van de Walle (2004) argues that when learners are only taught the performance of the skills of a particular procedure at the expense of developing and engaging with the concept itself, they become reluctant to attach meaning to it. This problem poses many challenges for mathematics teachers.

In order to respond to this challenge, many educational researchers advocate the use of physical manipulatives in order to “build connections between mathematics concepts and representations, fostering more precise and richer understanding” (CITED, n.d.). Physical manipulatives are tangible artefacts that encourage a hands-on engagement with the topic under consideration, and which are specifically designed to foster learning in a teaching and learning environment (Zuckerman, Arida & Resnick, 2005).

In order for my learners to explore surface area and volume in a meaningful hands-on manner I designed a series of four activities that used physical manipulatives to gradually develop learners’ conceptual understanding of these two important concepts.

Activity 1

The first activity focused on determining the area of squares and rectangles. The activity aimed to support learners’ understanding that the term ‘area’ simply refers to the number of squares of a chosen size that cover a particular region/shape or which are enclosed within a specified boundary. Learners were presented with square grid paper which contained square or rectangular bounded regions (Figure 1).

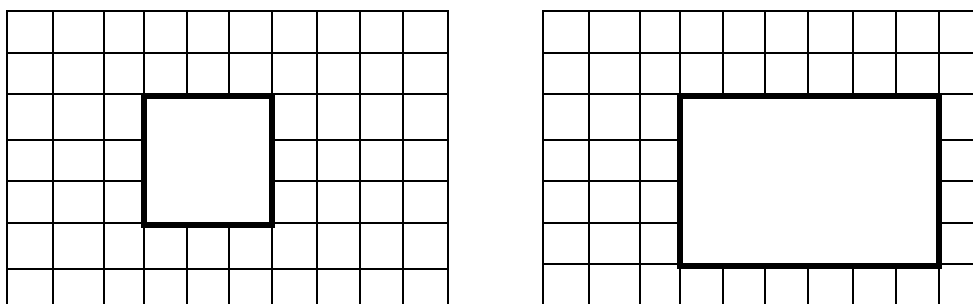


FIGURE 1: Grid paper containing square or rectangular bounded regions.

Learners were required to determine the area of each bounded region by identifying the number of unit squares contained within each region. To assist with this process, learners were given square tiles made from laminated paper that they could use to tile the enclosed regions. The emphasis of this process was for learners to share their different strategies for determining the number of unit squares in each region, with particular emphasis being placed on finding the most economical strategy – for example multiplying the number of squares in a single column by the number of identical columns, thereby forging a conceptual link to the conventional formula for determining the area of a rectangle: $\text{Area} = \text{length} \times \text{breadth}$.

Activity 2

This activity entailed designing nets for rectangular prisms, and subsequently constructing the 3-dimensional rectangular prisms from the nets (Figure 2). The various nets were constructed from cardboard on which a square grid had been carefully drawn. When the nets were folded to form the prisms, the folding was done in such a way as to ensure that the grid was on the outside of the shape. This provided a crucial conceptual link to the previous activity, and paved the way for learners to explore the concept of total surface area in Activity 3.

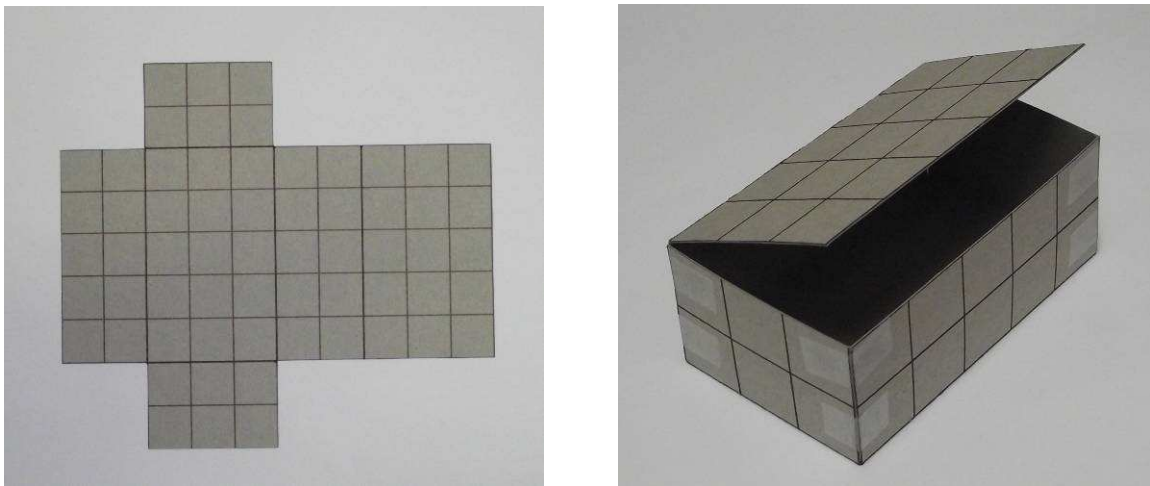


FIGURE 2: A rectangular prism formed from a net drawn on gridded cardboard.

Activity 3

This activity focused on determining the area of each of the six faces of the rectangular prisms constructed in Activity 2. In order to accomplish this, learners were encouraged to draw on their experiences from Activity 1 in which the concept of area was explored in terms of the number of unit squares contained within a bounded region. This conceptual link allowed learners to engage with the idea of total surface area, i.e. the number of unit squares on the surface of a given object.

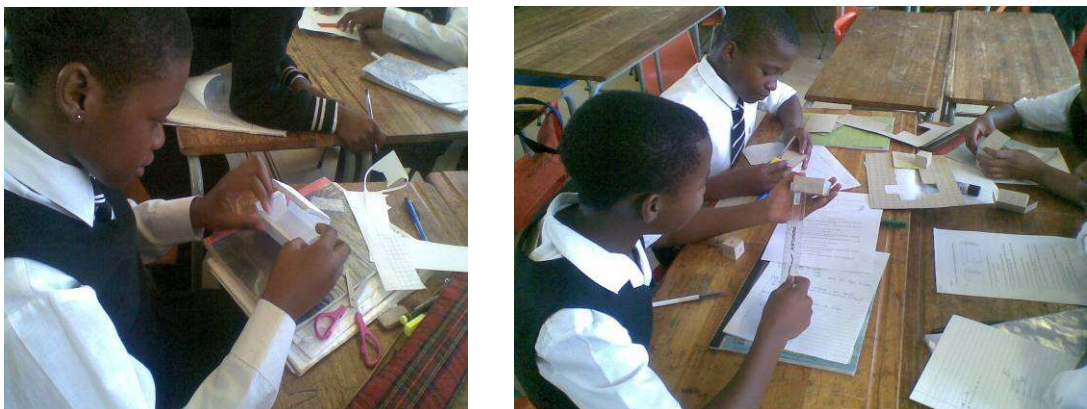


FIGURE 3: Learners determining the total surface area of rectangular prisms.

Activity 4

This activity moved the focus from surface area to volume, but was conceptually linked to the previous activities. In Activity 2 the nets that were designed for the rectangular prisms were constructed from cardboard on which a square grid had been carefully drawn. This square grid was drawn such that the size of the unit square contained in the grid exactly matched the dimensions of a large number of identical dice which had been purchased for Activity 4. Each face of the dice thus exactly matched the unit squares on the surface of the rectangular prisms, and the volume of these rectangular prisms could thus be explored by packing the prisms with unit cubes represented by the dice (Figure 4). The main aim of this task was to establish the idea that determining the volume of an object is all about finding the amount of space, expressed in terms of unit cubes, contained within the object.

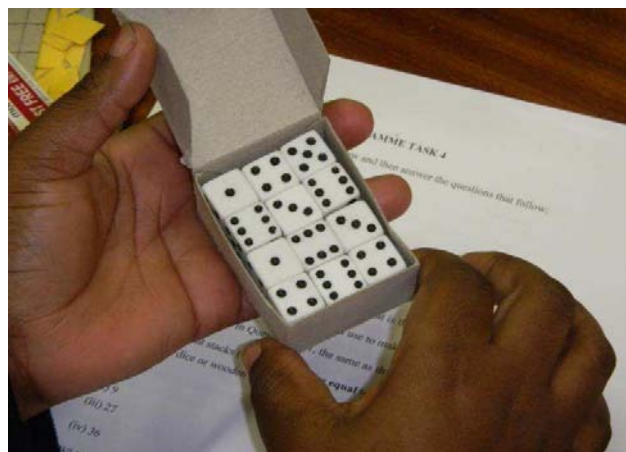


FIGURE 4: Determining the volume of a rectangular prism by packing it with unit cubes (dice).

Concluding comments

The four activities described in this article were carefully designed and sequenced to gradually develop learners' conceptual understanding of surface area and volume. The activities were designed to engage learners in meaningful hands-on tasks through the use of simple physical manipulatives. Although preparation for these tasks was time-consuming, and the process of moving through the sequence of tasks was slow for some learners, I believe the outcome was very encouraging. In the words of one learner, "*the use of physical manipulatives has taught us easy methods of calculating the total surface area and volume of prisms ... the learning of how to find the total surface area and volume of prisms has been simplified*". The use of simple physical manipulatives in the teaching and learning of area and volume proved successful in (i) providing a tangible context to what is for many learners an abstract concept, (ii) motivating learners through hands-on engagement, (iii) mediating learning by encouraging meaningful and contextualised learner-to-learner and teacher-to-learner conversation, and (iv) allowing learners to explore and experience the fundamental difference between the two important concepts of surface area and volume.

Acknowledgement

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