An Interesting Triangle Identity

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Consider an acute-angled triangle ABC with side lengths a, b and c as shown in Figure 1. A perpendicular is dropped from vertex A to point D on BC.

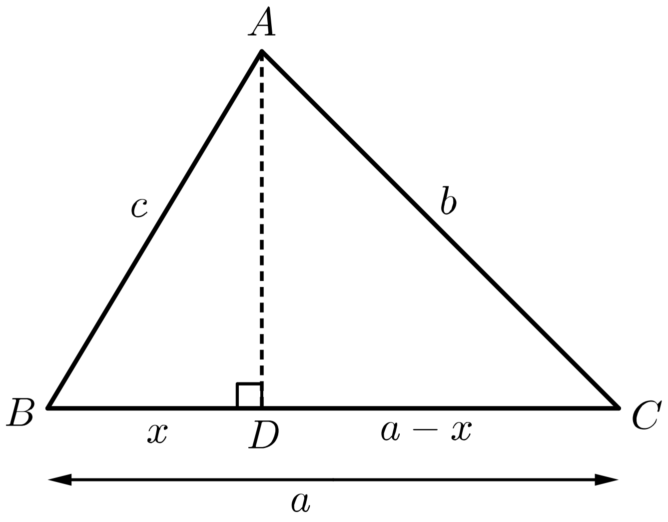

FIGURE 1: Triangle ABC with side lengths a, b and c.

In triangle ABD we have $\cos B = \frac{x}{c}$ from which $x = c.\cos B$. In triangle ACD we have $\cos C = \frac{a-x}{b}$ from which $x = a - b.\cos C$. Equating these two expressions for x and rearranging gives:

$$a = b.\cos C + c.\cos B$$

It can similarly be shown that:

$$b = a.\cos C + c.\cos A$$

$$c = a.\cos B + b.\cos A$$

We thus have:

$$a + b + c = b.\cos C + c.\cos B + a.\cos C + c.\cos A + a.\cos B + b.\cos A$$

This can be rearranged to give the following interesting result:

$$a + b + c = (b + c).\cos A + (a + c).\cos B + (a + b).\cos C$$

It is left to the reader to confirm that this result also holds true for an obtuse-angled triangle.