The Answer is 2024, But What is the Question?

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INTRODUCTION

The activities in this article all centre on the number 2024. Some feature 2024 as the answer or endpoint, while others are based on interesting properties of 2024. The activities are generally either open questions or problem-solving challenges suitable for various ages and learning stages. The activities can be structured in a way that allows for a differentiated approach in the classroom where learners can engage at different levels of complexity. The amount of scaffolding each task will require is dependent on the age group of the learners and their current mathematical attainment. Teachers should provide appropriate guidance as necessary.

ACTIVITY 1: CALCULATIONS THAT HAVE 2024 AS THE ANSWER

2024 has some interesting properties and features. Pique your learners' curiosity with the following interesting calculations:

$$(20 + 24) + (20 + 24)(20 + 24) + (20 + 24) = ?$$

$$2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 = ?$$

$$2^2 + 16^2 + 42^2 = ?$$

$$5^1 + 7^2 + 6^3 + 1^4 + 4^5 + 3^6 = ?$$

Now challenge learners to come up with their own questions with the answer 2024. Use this activity to encourage learners to think for themselves and to use everything they know about numbers. Perhaps ask your learners to come up with an easy question, a harder one, and then one that is very hard. This can be a very informative activity for assessing what understanding of number operations and what number sense and creativity the individual learners have. It is a good activity for the start of the school year and can obviously be adapted year by year. This activity allows learners to be creative, to explore and play with numbers and to come up with their own mathematical ideas.

You might like to use the 'One-Two-Four-More' strategy getting the learners to work individually until each learner has a calculation that gives the answer 2024. Then tell the learners to work in pairs to check that their partner's calculation does indeed have the answer 2024. Then ask the learners to work in fours. Perhaps each group could make a poster showing some calculations that have the answer 2024. Ask each group to contribute one of their 'calculations' to a class pool. You could then have a class vote on which question is the 'most interesting'. This could then lead to a class discussion as to what might constitute an 'interesting question' in this context.

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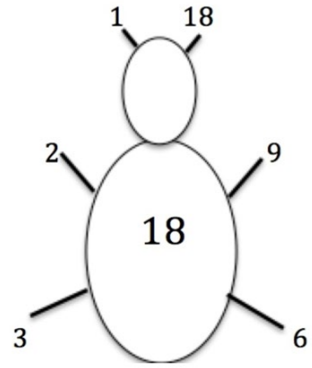
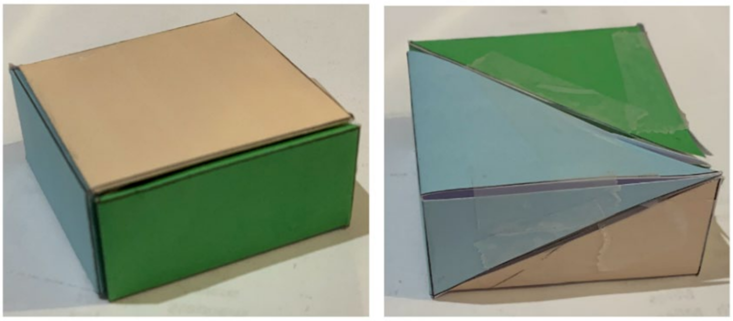
ACTIVITY 2: INVESTIGATING THE FACTORS OF 2024

Factor bugs are a useful way to visualise the factors as well as the factor pairs of a number. The factor bug for 18 is shown alongside. The antennae show that $1 \times 18 = 18$, and the pairs of legs show that $2 \times 9 = 18$ and $3 \times 6 = 18$. The more legs the factor bug has, the more factors it has. Now get your learners to draw the factor bug for 2024. How many legs does it have? Once learners are happy with their factor bug they then can then move on to prime factorising 2024.

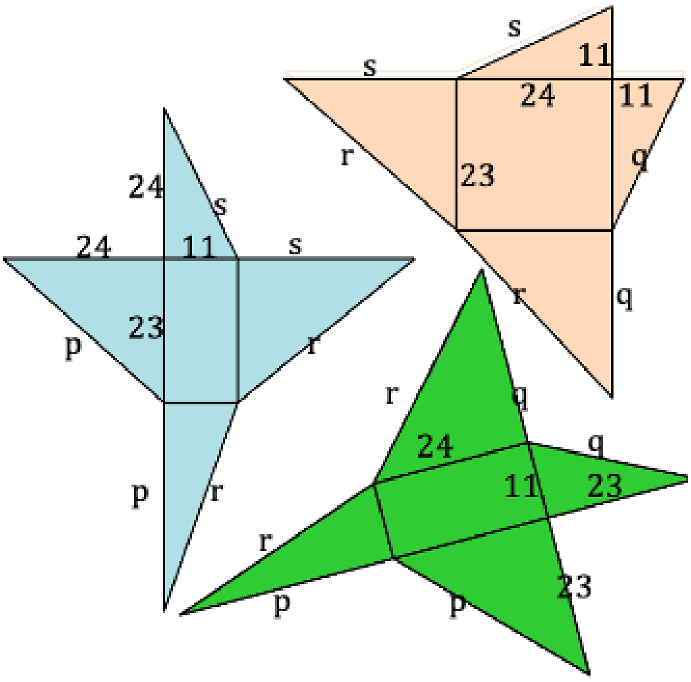


ACTIVITY 3: BUILDING A CUBOID (RECTANGULAR PRISM) FROM THREE RIGHT PYRAMIDS

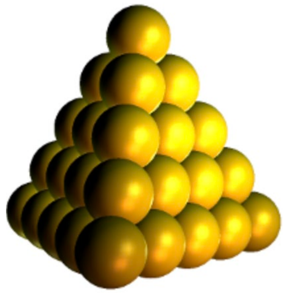
2024 can be prime factorised as $2^3 \times 11 \times 23$. Using this prime factorisation the challenge is for learners to find the dimensions of a cuboid (with integer side lengths) that can be split into three right pyramids (illustrated below) each having a volume of 2024 units^3. Although the three right pyramids don't have to be identical, they must each have the same volume (2024 units^3).



The activity hinges on the formula for the volume of a right pyramid being $\frac{1}{3} \times area\ of\ base \times height$. Nets for the three pyramids, with the necessary measurements, are illustrated below. As an alternative activity, learners can draw these nets to scale and then construct the three right pyramids. They can then calculate the volume of each pyramid and then try to fit them together to form the cuboid. Learners could also calculate the side lengths p, q, r and s using the Theorem of Pythagoras.

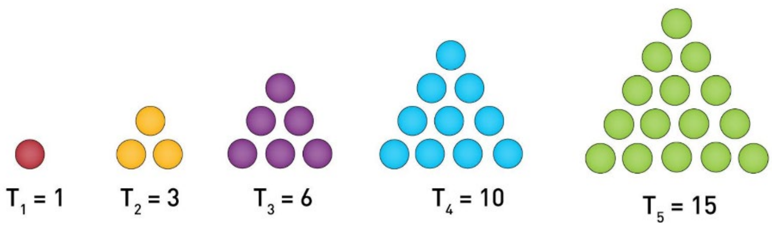


ACTIVITY 4: INVESTIGATING 2024 AS A TETRAHEDRAL NUMBER



2024 is the 22nd tetrahedral number and can be expressed as $\frac{22 \times 23 \times 24}{6}$. The last year that was a tetrahedral number was 1771, and it will be 276 years until the next one in 2300. Tetrahedral numbers can be represented visually by stacking spheres in triangular layers. The stack of 35 spheres shown alongside has five triangular layers and is the 5th tetrahedral number.

The number of spheres in each layer are the triangular numbers. For the 5th tetrahedral number illustrated above, the bottom layer has 15 spheres, the next one up has 10 spheres, the next layer has 6 spheres, then 3 spheres, and then a single sphere on the top layer. For a total of 2024 spheres, exactly 22 layers are needed.



Since each layer of the tetrahedral structure is a triangular number, the nth tetrahedral number is simply the sum of the first n triangular numbers:

n	nth Triangular number	nth Tetrahedral number
1	1	1
2	$1 + 2 = 3$	$1 + 3 = 4$
3	$1 + 2 + 3 = 6$	$1 + 3 + 6 = 10$
4	$1 + 2 + 3 + 4 = 10$	$1 + 3 + 6 + 10 = 20$
5	$1 + 2 + 3 + 4 + 5 = 15$	$1 + 3 + 6 + 10 + 15 = 35$
6	$1 + 2 + 3 + 4 + 5 + 6 = 21$	$1 + 3 + 6 + 10 + 15 + 21 = 56$

Begin this activity by physically building tetrahedral numbers by stacking items in layers that form triangular numbers. These items don't necessarily have to be spheres. After introducing the idea of triangular and tetrahedral numbers, challenge your learners to establish whether or not 2024 is a tetrahedral number. For younger learners you might need to scaffold the activity a bit in terms of structure. For older learners, challenge them to find a general formula for the nth triangular number as well as the nth tetrahedral number.

ACTIVITY 5: WRITING 2024 AS THE SUM OF POWERS OF WHOLE NUMBERS

2024 cannot be written as the sum of two squares. However, it can be written as the sum of three squares in seven different ways. By way of example:

$$2^2 + 16^2 + 42^2 = 2024$$

All natural numbers can be written as the sum of four squares – this is known as Lagrange's four-square theorem or Bachet's conjecture, and was proved by Joseph Louis Lagrange in 1770. 2024 can be written as the sum of four squares in 16 different ways. For example:

$$2^2 + 18^2 + 20^2 + 36^2 = 2024$$

If your learners know how to do simple coding then they should be able to adapt the pseudo-code below to whatever language they know to find different ways of writing 2024 as sums of squares and then adapt it again to find 2024 as sums of cubes, or any other powers. The pseudo-code and the results are given in the table below.

<table>
<tr>
<td>Pseudo-code to find sums of squares that give the answer 2024

The number 2024 cannot be written as the sum of 2 squares. The program below is written so that $c \geq b \geq a$. It checks values up to the square root of 2024, so it checks numbers from 1 to 45.</td>
<td colspan="3">The sum of the squares of 7 sets of 3 numbers all add up to 2024.

The sum of the squares of 16 sets of 4 numbers all add up to 2024.</td>
</tr>
<tr>
<td>for a=(1:45)
for b=(a:45)   as $b \geq a$
for c=(b:45)   as $c \geq b$
$x = a$^2+b^2+c^2;   checks sum of squares
if x == 2024   If sum is 2024 prints a,
disp([a,b,c])   b and c. If not goes on to
end end end end   next values.</td>
<td>2 16 42
2 24 38
8 14 42
10 18 40
10 30 32
16 18 38
18 26 32</td>
<td>2 18 20 36
4 6 6 44
4 6 26 36
4 10 12 42
4 18 28 30
6 8 18 40
6 8 30 32
6 12 20 38</td>
<td>6 16 24 34
8 22 24 30
10 12 22 36
10 18 24 32
12 14 28 30
12 18 20 34
14 24 24 26
18 20 20 30</td>
</tr>
</table>

There are many other interesting properties of 2024. For example, there are only two lists of numbers such that $a^1 + b^2 + c^3 + d^4 + e^5 = 2024$. One is $6^1 + 3^2 + 9^3 + 4^4 + 4^5$. Use the pseudo-code alongside to try to find the other one. What other interesting properties of 2024 can you find by adapting the code?

```
for a=(1:10)
for b=(1:10)
for c=(1:10)
for d=(1:10)
for e=(1:10)
if a + b^2 + c^3 + d^4 +e^5 == 2024
x = [a,b,c,d,e];
 disp(x)
end end end end end end
```

CONCLUDING COMMENTS

There are many other activities that can be structured around the number 2024. For example, how many different 4-digit numbers can be created by using the four digits in 2024? If you take 2024 and add it to 4202 (i.e. the number created by reversing the digits of 2024) the answer is the palindromic number 6226. Under what conditions will this kind of sum yield a palindromic answer? This article will hopefully inspire you to explore other possible mathematical activities based on 2024.

More information about these and other related activities can be found on the AIMSSEC[2] Teacher Network site https://aiminghigh.aimssec.ac.za that offers a wealth of free lesson resources under creative commons.

[2] AIMSSEC is a not-for-profit organisation that aims to empower teachers from disadvantaged rural and township communities in South Africa and other parts of the world. AIMSSEC offers professional development courses (both residential as well as online) aimed at enhancing teachers' subject content, pedagogical content and technological content knowledge. The courses are run in collaboration with a team of local, continental and international professional mathematics educators. In South Africa, the courses are fully funded for teachers and leaders of mathematics.