Making the Case for More Modelling in Mathematical Literacy (and Mathematics!)

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'MATHS FOR DUMMIES'

The subject Mathematical Literacy (ML) often gets bad press, and some influential people like Jonathan Jansen have made their negative feelings about this subject explicitly clear (e.g. Smith, 20 June 2019). A key component of these criticisms is commonly linked to the lower level of mathematics in ML when compared to the subject Mathematics, with ML sometimes referred to as 'Maths for Dummies' or a 'second-rate maths'. Focusing on the mathematics is, for me, problematic for two reasons. First, the subject was never intended to provide students with learning experiences focused on abstract and complex mathematical concepts, since this is what Mathematics is for. How ironic that a subject designed for students who traditionally would have opted out of continuing to study Mathematics, often because they found Mathematics too difficult, is criticised because of a lower level of mathematics! Second, focusing on the mathematics diverts attention away from the actual purpose of the subject; namely, to engage in real-world problem-solving experiences to better prepare students to participate critically and in an informed way in current and future real-life and workplace practices. In other words, for empowered life-preparedness (North, 2017).

KEY ELEMENTS OF WHAT IT MEANS TO BE MATHEMATICALLY LITERATE

Achieving this intended purpose requires a number of elements in ML classrooms and assessments (all of which are embedded in the ML CAPS curriculum (DBE, 2011) – see North (2024 - under review) for an analysis of these elements). The elements include:

- working with **authentic real-world situations and resources** as opposed to resources that have been overly simplified or adapted to foreground specific mathematical methods or concepts. But, at the same time, also realising that any analysis of a real-world context in a classroom will ever only be able to provide a limited and simulated experience of that context;
- working with **genuine problems** that might be encountered by people who operate in those contexts on a daily basis;
- **drawing on a range of tools and information, including some mathematics, technology, and context-specific meanings** to make sense of, model, and solve these genuine problems;
- recognising that although generalisable mathematical methods can be helpful for modelling these problems, **people also use a range of informal and context-specific methods and techniques**. These other methods should be given equal consideration;
- recognising that the mathematical solution does not always provide the most helpful solution and **consideration must be given to contextually specific constraints** that may influence the choices people make;
- acknowledging that **most daily life practices do not require engagement with complex abstract mathematics** (and, also, that an understanding of complex mathematics does not directly correlate to more empowered and informed decision making in real-world practices);
- **communicating ideas and decisions effectively**, using both mathematical and contextually appropriate tools.

Taken together, these elements frame a specific vision for what it means to be 'mathematically literate'. Namely, that being mathematically literate in a South African context does not mean being able to do or understand lots of maths. Rather, it means being able to draw on a range of tools and resources, including some mathematics, some technological tools, and some contextual knowledge, to make sense of and solve problems encountered in any real-life situation in order to facilitate empowered and informed decision-making. Examples of this type of informed decision making include: whether a specific loan option is a better deal; the potential consequences of being on strike for too long; and the appropriate quantity of materials needed to complete a project. For me, it is this vision of mathematically literate behaviour that has the potential to facilitate empowered and enriching learning experiences for learners in the subject ML for their current and future lives.

A FOCUS ON SKILLS RATHER THAN MATHEMATICAL CONTENT

Framing what it means to be mathematically literate in this way signals a shift in focus in the envisioned outcome of a learning experience in the subject ML. In Mathematics, learners engage with more and more mathematics, and as they progress through each grade the mathematics content becomes more complex. In ML, by contrast, it would be impossible for learners to learn about all of the contexts they might encounter in their current and future lives. Equally, they also potentially won't learn more mathematics since much of the mathematical content needed in ML has already been covered in previous grades. Instead, in ML the aim is for learners to develop a set of *skills* that will enable them to become increasingly confident and independent at identifying, finding, selecting, and using a range of tools and understandings (mathematical, technological and contextual) to interpret, decode, model, and solve any problems that they may encounter in both familiar and unfamiliar contexts. These skills include: fluency with mathematical concepts and tools; estimating efficiently and measuring with precision; application and modelling to enable problem-solving; interpretation and comprehension; critical reasoning and reflection; communication; and an inquiring disposition. When we think about ML in this way, then it is so much more than just 'watered down maths' because, as we all know from first-hand experiences, solving real-world problems that comprise multiple variables is a complex, challenging and demanding endeavour.

For the rest of the article I want to focus specifically on one skill, that of *modelling*. For me, modelling is an essential trait of a mathematically literate individual who is able to make informed and calculated decisions in their everyday lives; and I remain convinced that learners and adults who have the skills to generate models of real-world problems are in a more empowered position to make informed decisions than those who can't. Unfortunately, this skill seems to be somewhat overlooked in some ML classroom and assessment practices (and also in Mathematics), and this article attempts to re-shine a light on this aspect of empowered mathematically literate behaviour.

MODELLING – SOME BRIEF THEORY

By modelling I am referring to the process of organising, representing and solving a real-world problem using a combination of mathematical contents and methods and technological tools. Figure 1 shows one example of a modelling cycle and of the components and processes involved in modelling activities.

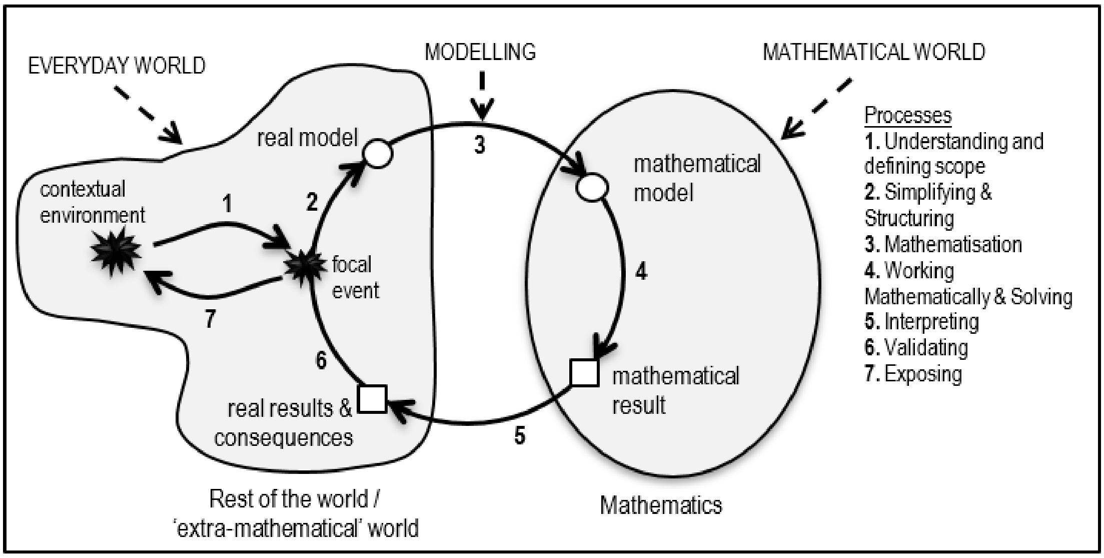


FIGURE 1: Modelling cycle (adapted from Blum & Leiß, 2007)

As shown in the figure, the type of modelling that I am proposing for ML starts with a problem situation – a focal event (e.g. contract vs. pre-paid cell phone costs for a specific company) – embedded in a broader real-world contextual environment (e.g. telephone costs and tariffs). It is in relation to this specific focal event that a model is deemed helpful and necessary for describing and understanding the problem situation. Model building is only possible with a clear (1) understanding of the problem situation (e.g. specific cell phone tariffs; other influences on costs, such as free minutes), of the embedded nature of the problem within a broader contextual environment (e.g. other companies may offer different options and tariffs), and, consequently, of considerations in the wider environment that might impact on how to interact with the problem situation (e.g. irrespective of the most cost effective option, the lack of availability of certain phone models may affect people's decisions).

Understanding of the problem is followed by (2) simplification and structuring of the problem into a 'real model' to more clearly define and focus the criterion of the problem situation and to make the problem more accessible to the problem solver based on their current knowledge or experience of the situation (e.g. choosing to compare costs for voice calls, data and local SMS's only and ignoring international usage). This simplified real model is then (3) mathematised and (re)organised and (re)structured around mathematical principles in conjunction with technological tools to produce a more mathematical or calculation-based model (e.g. drawing graphs to compare options), which is then (4) solved to produce a more mathematical-focused result (e.g. identifying the most cost-effective option based on cost-calculations only). Note that during the formulation of the real (2) and mathematical (3) models, deliberate decisions and selections are made about which variables to include and which to ignore in the model (e.g. choosing to focus on only one contract type and one pre-paid tariff for one company). These selections will directly influence our understanding of the problem and may also influence any decisions made from interpretations of the model. In other words, the model provides a limited and situated perspective of the problem, and it is essential to remember that in most cases the model can never fully capture all variables and considerations of the contextual environment.

Since the main purpose for constructing the model is to facilitate greater understanding of the real-world problem to support informed decision-making, the mathematical result must be re-translated into the contextual environment. This involves: (5) interpreting and (6) validating the model and its solution in relation to the consequences, criteria, constraints, and everyday considerations that affect decision-making in that real-world situation: just because the model presents a particular way of thinking about and understanding the problem mathematically doesn't mean that this is the optimal or most feasible solution under a certain set of contextual constraints (e.g. if the cash price of the phone is too high then the pre-paid option may not be feasible even if it is more cost effective).

The final step is for the model to be (7) exposed, compared and reflected against the broader contextual environment to see whether the model still provides a helpful description of the problem when broader influences and considerations are factored in (e.g. will a specific option still be the most cost effective when compared with other options and other phone companies?).

In the next section I illustrate this modelling approach in relation to the specific real-world problem of comparing cell phone costs.

A MODELLING EXAMPLE: WHICH CELL PHONE OPTION?

According to the 2022 national census, 92,1% of all South African's own a cell phone. As such, making decisions regarding cell phone options is a scenario that most people face and will continue to face on a recurring basis in their daily lives.

The pictures below show two different options from the same company for the same cell phone (*process 1*) – contract on the left and pre-paid on the right.

FIGURE 2: Contract and pre-paid cell phone adverts

Apologies to readers if this is common knowledge, but to ensure consistency of understanding of the context (*process 1*) it is necessary to establish the following:

- for the pre-paid option (right), you buy the phone outright and then buy vouchers to be able to use the phone to make calls and connect to the internet. This pre-paid option does not include an allowance of free minutes or data;
- for the contract option (left) there is a fixed monthly fee of R259,00 payable each month for a total of 24 months. This fee covers the cost of the phone and the cost of the included free minutes (100) and data (512MB Anytime Data and 512MB Social Data for Twitter, etc.).

Ignoring, temporarily, considerations that may make either of these options unfeasible for some people (*process 2*) (e.g. the purchase price of the pre-paid option; or the recurring monthly cost on the contract option), one question that could be asked of this scenario is:

Which option is the most cost-effective for a person with specific cell phone usage needs?

Below I illustrate the development of a model that makes it possible to begin to frame an answer to this question.

MAKING SELECTIVE CHOICES

One option for a model to address this question would be to explore the relative monthly cost of each option for different usage amounts. This, in turn, necessitates information about the usage costs for these pre-paid[3] and contract[4] options. But, since for this company there are four possible pre-paid tariffs to choose from[5] and multiple contract options, it becomes necessary to make a selective decision about which tariffs to use as a starting point in the model (*process 2*) (the model could be adjusted later to accommodate other tariffs). So, for purposes of my cost comparison model, the comparison will draw on the selected tariffs shown in the tables below.

Cell C Elevate 1	
Cell C Elevate Social Data	512 MB
Monthly Fee	R 259.00
Anytime Data	512 MB
Minutes	100 Any-net
Data Per MB (Out of Bundle)	R 0.40
Local SMS Per Message	R 0.50
Local Voice Per Min (Out of Bundle)	R 0.99

Cell C 4Eva Rates

Type	Rate
Data (any time of day)	R 0.06 per MB
Voice (any network at any time of day)	R 0.49 per minute
SMS (any local network at any time of day)	R 0.29
MMS (any local network at any time of day)	R 0.50
International SMS	R 1.50
International MMS	R 1.50

It is also necessary to make a selective decision about which usage types to focus on in the comparison, and the model shown below prioritises only *voice calls*, *local SMS's* and *data*. A further consideration is how to incorporate the purchase price of the pre-paid phone in the cost comparison to ensure a more accurate comparison of all costs on the contract and pre-paid options. As such, in the model shown in Figure 3, a deliberate decision was made to reflect the pre-paid phone purchase price as a comparative *monthly cost* relative to the 24-month length of the contract option.

BUILDING THE MODEL AND USING APPROPRIATE TECHNOLOGIES FOR AUTOMATICITY AND DYNAMICITY

Having decided on a specific focal event and the specific variables for inclusion and comparison in that event, the next step is to build a suitable mathematically based model (*process 3*) that will facilitate a cost comparison. In this context and problem scenario, a key consideration is how changes in usage amounts for three variables (voice calls, SMS's and data) affect the costs on each option.

Given the power of technology for *automating* calculations and showing changes in relationships *dynamically*, a deliberate decision is made to use a spreadsheet (Figure 3) to structure and automate this cost comparison model. Explanations of specific components of the model are provided below the figure. Scan the QR code alongside or use the web link https://bit.ly/42LiglS to download this interactive spreadsheet model. You will need to open the spreadsheet in the *desktop version of Excel* to make use of the interactive sliders.

[3] https://www.cellc.co.za/cellc/prepaid-contract-detail/for-eva

[4] https://www.cellc.co.za/cellc/data-contract-detail/elevate#!#sku14121273919

[5] https://www.cellc.co.za/cellc/prepaid-contracts

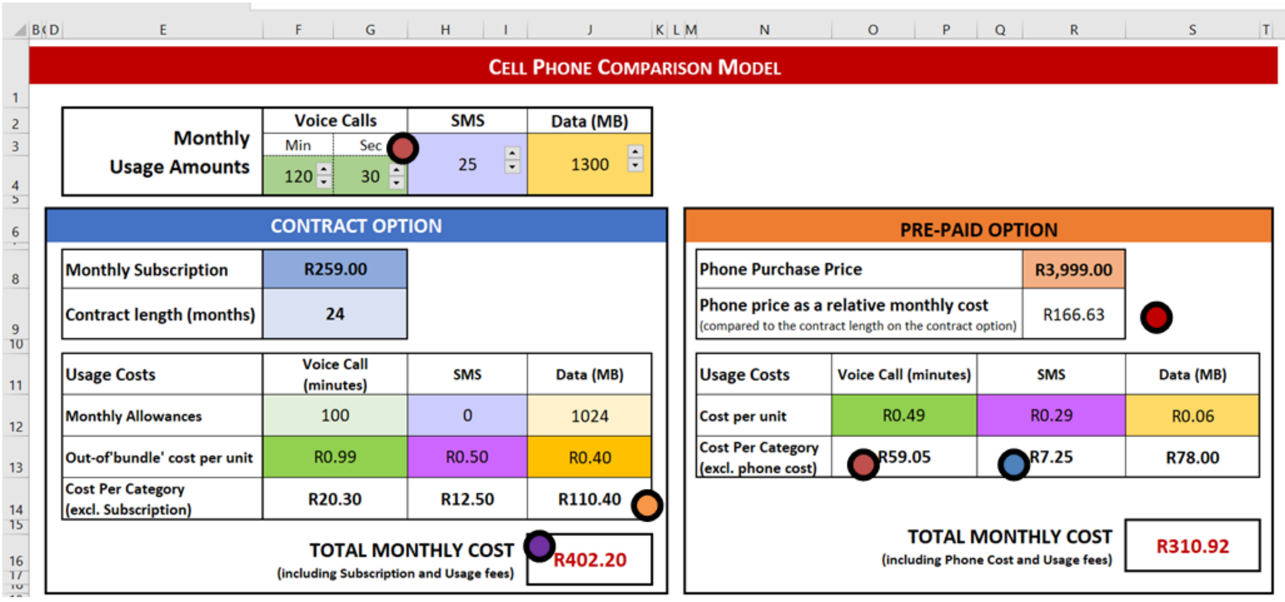


FIGURE 3: Spreadsheet model of the phone cost comparison

Explanations of specific aspects of the spreadsheet cost comparison model		
●	Although there is no monthly subscription on the pre-paid option, we need to account for the initial purchase price of the phone. This calculation allows for a more accurate comparison of all costs on the pre-paid option with all costs on the contract.	Phone price as a relative monthly cost = Phone Purchase Price ÷ 24 (contract length) = R3 999,00 ÷ 24 = R166,63
●	To work out a 'Cost Per Category' value, you multiply the 'Usage Amount' by the 'Cost/unit'.	e.g. Monthly cost of SMS's on the pre-paid option = 25 SMS × R0,29/SMS = R7,25
●	On the contract, a usage charge only applies for calls/data that exceed the free 'monthly allowances'.	e.g. 1 300 MB of data is used, which exceeds the monthly data allowance by 1 300 – 1024 = 276 MB → Cost of 'out-of-bundle' data = 276 MB × R0,40/MB = R110,40
●	The charge for calls on both options is *per minute* and any 'seconds' values need to be converted into minutes before being multiplied by the per minute charge.	e.g. Total call time = 120 m 30 s = 120 min + (30 ÷ 60) min = 120,5 min Total pre-paid call costs = 120,5 min × R0,49/min = R59,05
●	The 'Total Monthly Cost' value is the sum of the usage charges (voice, data, SMS) plus any other charges like the subscription fee or relative monthly cost of the pre-paid phone.	e.g. Total monthly cost on the contract option = Subscription + 'Out-of-bundle' usage charges = R259,00 + R20,30 (voice) + R12,50 (SMS) + R110,40 (data) = R402,20

By entering values manually, or by using the sliders, we can explore dynamically how the monthly cost on each option varies and changes for different combinations of usage values. We can also explore the impact of the 'free allowance' allocations on the contract option and whether and how this affects the price comparison (*processes 3 & 4*).

Monthly Usage Amounts	Voice Calls Min	Voice Calls Sec	SMS	Data (MB)
	50	12	10	512

Usage values within the 'free allowance' allocation for voice calls and data on the contract

CONTRACT OPTION

Monthly Subscription	R259.00
Contract length (months)	24

Usage Costs	Voice Call (minutes)	SMS	Data (MB)
Monthly Allowances	100	0	1024
Out-of'bundle' cost per unit	R0.99	R0.50	R0.40
Cost Per Category (excl. Subscription)	R0.00	R5.00	R0.00

TOTAL MONTHLY COST (including Subscription and Usage fees): R264.00

PRE-PAID OPTION

Phone Purchase Price	R3,999.00
Phone price as a relative monthly cost (compared to the contract length on the contract option)	R166.63

Usage Costs	Voice Call (minutes)	SMS	Data (MB)
Cost per unit	R0.49	R0.29	R0.06
Cost Per Category (excl. phone cost)	R24.60	R2.90	R30.72

TOTAL MONTHLY COST (including Phone Cost and Usage fees): R224.84

Marginally cheaper option

Monthly Usage Amounts	Voice Calls Min	Voice Calls Sec	SMS	Data (MB)
	150	46	40	2000

Usage values exceed the 'free allowance' allocation for voice calls and data on the contract

CONTRACT OPTION

Monthly Subscription	R259.00
Contract length (months)	24

Usage Costs	Voice Call (minutes)	SMS	Data (MB)
Monthly Allowances	100	0	1024
Out-of'bundle' cost per unit	R0.99	R0.50	R0.40
Cost Per Category (excl. Subscription)	R50.26	R20.00	R390.40

TOTAL MONTHLY COST (including Subscription and Usage fees): R719.66

PRE-PAID OPTION

Phone Purchase Price	R3,999.00
Phone price as a relative monthly cost (compared to the contract length on the contract option)	R166.63

Usage Costs	Voice Call (minutes)	SMS	Data (MB)
Cost per unit	R0.49	R0.29	R0.06
Cost Per Category (excl. phone cost)	R73.88	R11.60	R120.00

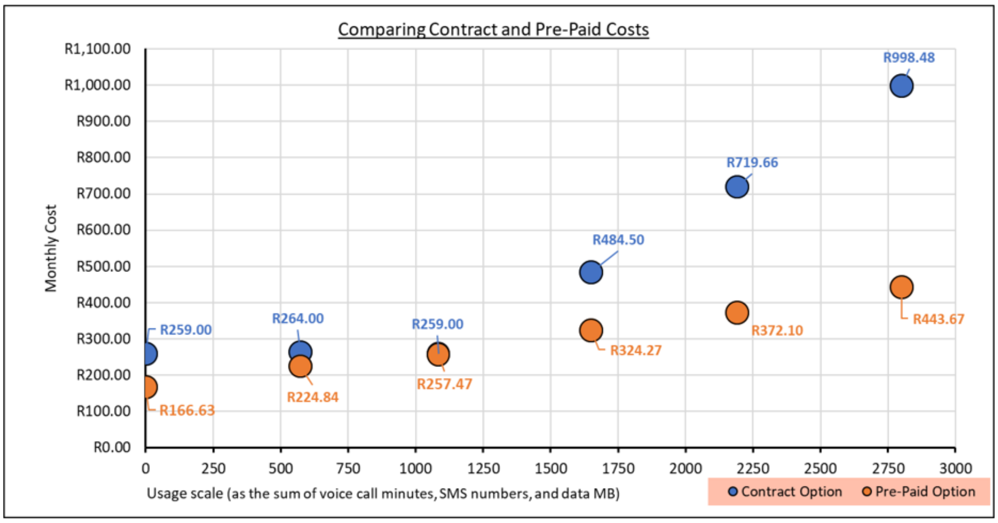
TOTAL MONTHLY COST (including Phone Cost and Usage fees): R372.10

Significantly cheaper option

FIGURE 4: Cost comparison on each option for different usage amounts

Another powerful component of technology for supporting modelling activities is the ease with which graphs can be generated (*process 3*). In addition, these graphs can be developed to show *dynamically* how changes in usage amounts affect changes in the overall costs for each option. The hyperlinked spreadsheet model will give the reader a sense of this 'dynamic' property.

There are three different variables at play here - voice call usage, SMS usage and data usage. So, in order to draw the graph (Figure 5), I had to make a deliberate decision (*process 3*) about what variable to represent on the horizontal axis. I opted to combine all of the usage amounts into a single value by adding them together (voice calls in minutes + SMS amount + data amount), which I have called the '<u>usage scale</u>'. My decision to do this has affected the graphs that are produced, which in turn will influence my interpretation of the cost comparison for each option (*processes 4, 5 & 6*). Using a different scale or variable on the horizontal axis may yield a different analysis. In other words, and as mentioned before, the model offers a limited perspective of this real-world problem.



Usage amounts for each point							
		Point 1	Point 2	Point 3	Point 4	Point 5	Point 6
Voice	Min	0	50	60	120	150	200
	Sec	0	12	0	18	46	5
SMS		0	10	0	30	40	100
Data MB		0	512	1024	1500	2000	2500
'Usage Scale' value		0	≈ 572	≈ 1 084	1 650,3	≈ 219	≈ 2 800

Usage within free allowances on contract (Points 2–3) | Usage exceeds free allowances on contract (Points 4–6)

FIGURE 5: Graph and table showing a cost comparison of each option for different usage amounts

MAKING INFORMED CHOICES ABOUT THE MOST COST-EFFECTIVE OPTION … BUT …

Having the model available makes it possible to make an informed decision (*process 5*) about whether the contract or pre-paid option is the most cost effective for a given set of conditions. What the model clearly shows is that contract option is *always* more expensive than the pre-paid option even when the usage amounts fall within the 'free allowances' for calls and data and particularly when usage amounts exceed the allowances. The only instance where the costs of the two options are almost the same (Point 3) is when the usage amounts for the contract fall within the free allowances and there is no SMS usage.

But, despite what the 'mathematics' of this model tells us about the most cost-effective option, there are a number of real-world considerations that might influence a person to opt for the contract option even if they know that it is more expensive (*process 6*). The most obvious consideration is that many people may not have R4 000,00 available in cash to pay for the phone upfront on the pre-paid option, in which case this option isn't viable no matter how much more expensive a contract may be. This disconnect between mathematical solutions and real-world considerations is precisely why interpreting and reflecting on the validity of the model in relation to the real-world contextual environment (*processes 5 and 6*) are essential components of modelling activities. In addition, it's also essential to recognise that this model provides an incredibly limited perspective of cost comparisons of pre-paid and contract cell phone options. The model has only captured cost comparisons for *one* pre-paid scenario and *one* contract scenario from *one* company; there are a multitude of alternative possible tariff options and scenarios for this same company, not to mention the many other options available with different companies (*process 7*). As such, any interpretations, reflections, and decisions made may not be generalisable to other comparison scenarios.

Despite the limited perspective afforded by the model, being able to generate the model is empowering precisely because it allows the user of the model to make an *informed* decision – even if this informed decision is to choose an option that from a mathematical perspective is not the optimal option. Rather than having to rely on or be swayed by the advice or opinions of others, a mathematically literate individual is able to make these decisions for themselves, empowered by the capacity to generate simple models that draw on a combination of mathematics, technology, and contextual understanding to organise, simulate and decode real-world problems.

REALLY 'MATHS FOR DUMMIES' OR COMPLEX HIGHER ORDER REASONING?

Returning to the perception of ML as 'maths for dummies', if you focus solely on the mathematics in this problem then you might well critique the subject as being easy because, in truth, none of the mathematical calculations in this spreadsheet model are complex and none involve high levels of abstract mathematics. Only basic arithmetic skills are needed, including combinations of multiplication, subtraction, addition and division, plus time conversions. However, and this is a big however, if modelling is foregrounded as an essential skill of a mathematically literate learner, then the type of modelling activity illustrated above contains a significant degree of complex higher order thinking, reasoning and understanding. This includes engaging in critical reasoning about: which information and variables to include (and exclude) from the model; what mathematical contents and procedures to use; what technological and contextual tools to use; how to get the different elements of the model to interact with each other; how to present and package the model in an appropriate and accessible way; how to error check the accuracy of model; how to identify the limitations of the model; how to interpret the fidelity of the model to real-world practice; how to use the model to inform decision-making; and, how to communicate about your model to others so that they can understand and use the model. All of these require complex higher order thinking and reasoning skills combined with a deep understanding of the problem and the context.

PREPARING LEARNERS FOR A 21ST CENTURY WORLD

The 21st century world is a technology driven world, characterised by rich and complex problems involving multiple variables, intricate data, and sophisticated choices. If we want our ML (and Mathematics) learners to be empowered for this world, we need to equip them with the skills to solve complex problems and make informed decisions in that world. Giving students realistic modelling experiences with authentic problems and supporting them to develop modelling skills takes a positive step towards this empowerment goal.

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