Creativity with Areas – Circles and Squares

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INTRODUCTION

Fostering creativity in our pupils is essential if they are to successfully engage with the dynamic and rapidly changing world around us. Working with shape and space is one area of the curriculum where one can potentially encourage creative thinking through more open-ended activities and investigations. In junior grades, one of the goals of working with space and shape is the fostering of geometrical insight through the exploration of shapes, their properties and interrelations. In addition to fundamental properties such as area and perimeter, pupils should develop their visual and spatial perception skills in order to make the transition from basic to more complex shapes.

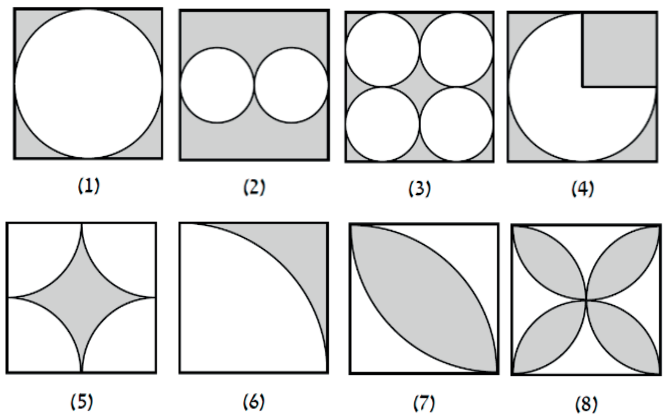
One way to increase students' learning motivation is through experiential learning. The activities described in this article aim to enable pupils to engage meaningfully with area, analyse a variety of complex shapes, identify useful sub-structures, and to use these sub-structures in the calculation of area using different approaches as well as the creative synthesis of new shapes. While pupils are engaged with these activities they should be encouraged to verbalise their thinking – either through discussion with a partner, or through a written articulation of their calculation strategy.

The two basic shapes focused on in this article are the square and the circle. Although simple in themselves, when used in combination they can result in a wide variety of complex shapes that can be decomposed in a variety of ways. This decomposition process should provide students with the necessary skills to generate new shapes, and determine their area, through combining fundamental components in novel ways.

ACTIVITY 1

The eight shapes that constitute this activity are presented in the given sequence so that pupils gradually advance their visual perception as the shapes become more complex. In addition, spatial insights gleaned from working with simpler shapes should be transferrable to more complex shapes.

This first activity focuses on the calculation of the area of the shaded region in each of the eight diagrams. For transferability, all eight diagrams are based on a large square with side length 20 mm. This is the only measurement given since all other necessary measurements can be deduced from it.



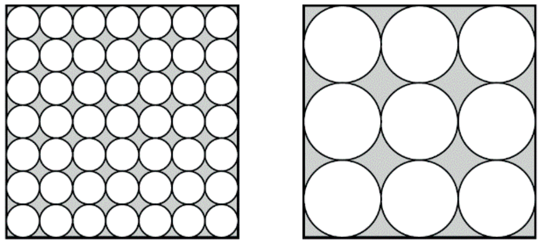
Area calculations

Far more important than the actual area calculation itself is the thought process behind the calculation. It is this aspect of engagement that this activity aims to foreground.

- **Shape 1**: The shaded area is simply the area of the square minus the area of the circle (r = 10 mm). Thus, area = $20^2 - \pi(10)^2 = 400 - 100\pi$ mm^2.
- **Shape 2**: The shaded area is the area of the square minus the area of two circles (r = 5 mm). Thus, area = $20^2 - 2 \times \pi(5)^2 = 400 - 50\pi$ mm^2.
- **Shape 3**: The shaded area is the area of the square minus the area of four circles (r = 5 mm). Thus, area = $20^2 - 4 \times \pi(5)^2 = 400 - 100\pi$ mm^2. Rather than doing the calculation from scratch, pupils should notice that all they need do is double the area being subtracted in shape 2 since the small circles are the same size.
- **Shape 4**: The shaded area is the area of the square minus three quarters of the area of the circle (r = 10 mm). Thus, area = $20^2 - \frac{3}{4} \times \pi(10)^2 = 400 - 75\pi$ mm^2. Pupils should also be encouraged to arrive at this answer in different ways – for example by taking three quarters of the shaded area of shape 1 and adding the area of a square of side 10 mm. Thus, area = $\frac{3}{4} \times (400 - 100\pi) + 10^2$ mm^2.
- **Shape 5**: The shaded area is the area of the square minus the area of four quarter circles (r = 10 mm). Pupils should hopefully come to the realisation that the area of four quarter circles is the same as the area of a whole circle. We thus have area = $400 - 100\pi$ mm^2. This is the same as the shaded area in shape 1. Pupils should be encouraged to visualise this by cutting shape 5 into four identical pieces that can be rearranged to form shape 1, or vice versa.
- **Shape 6**: The shaded area is the area of the square minus the area of a quarter circle (r = 20 mm). Thus, area = $20^2 - \frac{1}{4} \times \pi(20)^2 = 400 - 100\pi$ mm^2. Note that this is the same area as that of shape 1. An alternative method of calculating the area of shape 6 would be to take a quarter of the area of shape 1 (since this would be the same shape as that of shape 6) and then scale the area up by a factor of 4. This idea of identifying similar shapes and then scaling them appropriately is a useful strategy when working with shapes that have similar sub-components.
- **Shape 7**: One way of determining this shaded area is to determine the area of a quarter circle (r = 20 mm) and then subtract the area of the shaded region in shape 6. Alternatively, begin with a whole square and subtract two lots of the shaded region in shape 6. Area = $200\pi - 400$ mm^2.
- **Shape 8**: One way of determining the shaded area in shape 8 is to compare it to the shaded region in shape 7. Shape 8 can be seen to comprise four identical sub-structures each similar to shape 7, and each a quarter of the size. One can thus take the shaded area in shape 7, scale it down by a factor of 4, and then multiply it by 4. From this it becomes obvious that the shaded areas in shapes 7 and 8 are the same, namely $200\pi - 400$ mm^2. An alternative approach is to compare shape 8 with shape 5. If we subdivide shape 5 into four identical components by bisecting the square vertically and horizontally, it should become apparent that the area of the unshaded region in shape 8 is exactly twice the area of the shaded region in shape 5. We can thus determine the shaded region of shape 8 by doubling the shaded area of shape 5 and subtracting it from the square. We thus have area = $20^2 - 2(400 - 100\pi) = 200\pi - 400$ mm^2.

ACTIVITY 2

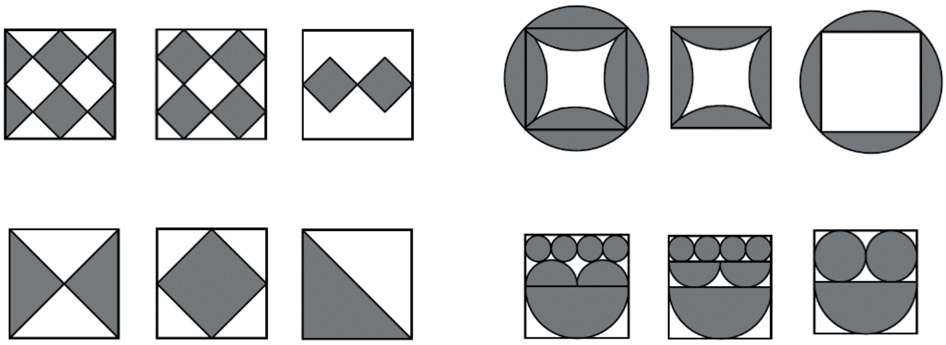
The second activity is more of an open-ended investigation. To begin with, two complex shapes are presented and pupils are asked to calculate, and compare, the shaded area of each shape. Once again the two squares are identical, each having side length 20 mm.



The result that the two shapes have the same shaded area is initially somewhat surprising. Furthermore, note that the area is the same as that of shape 1 and shape 3 in Activity 1. These observations should hopefully encourage pupils to explore similar scenarios – for example a square with an internal 4-by-4 or 5-by-5 array of circles. One could even try to generalise for an n-by-n array in order to explain why the shaded area remains constant.

ACTIVITY 3

In the third activity pupils are encouraged to create a series of related shapes by thinking of an initial design and then creating related shapes through translation, rotation, reduction etc. The only requirement is that all shapes are based on the basic shapes of a circle and a square, or sections thereof. In addition to creating the series of related shapes, students should also articulate how they might use the shaded areas of one shape to determine the shaded area of a related shape. Four sets of related shapes are shown below as examples:



CONCLUDING COMMENTS

The purpose of the activities presented in this article is to engage pupils in the exploration and calculation of related areas and the verbal articulation of their reasoning process. The activities are structured in a way so that pupils move from simple shapes to more complex related shapes, thereby gradually developing and refining their visual and special perception skills. In addition, creativity is encouraged and nurtured through open-ended activities involving related shapes. The activities also have the benefit of allowing differently able pupils to engage at different levels. While the activities presented here focused specifically on area, all three activities could equally be carried out with a focus on perimeter. Indeed, it is encouraged that both area and perimeter should be explored as this could lead to further insights and some interesting interrelations.