What is Your Philosophy of Mathematics?

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INTRODUCTION

The Curriculum and Assessment Policy Statement (CAPS) describes Mathematics as:

> …a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. (Department of Basic Education, 2011, p. 8)

The above description is multi-faceted, incorporating notions of Mathematics as a language of symbols; a human activity involving processes such as investigation and generalisation; a means of describing the physical and social world in which we find ourselves; a deductive system of abstraction; and a means of improving broader decision-making processes. The above description incorporates ideas from a variety of different philosophies of mathematics. However, as a teacher, what is your own philosophy of mathematics?

A QUICK QUIZ

Take a few minutes to answer the following six questions. Choose the letter (A, B, C or D) that most closely resonates with you in each case.

Do you think that mathematics is beautiful?

A	Yes, mathematical objects are aesthetically pleasing to me.
B	I like it when complex sums simplify to a clean, neat and elegant answer.
C	Isn't it awesome that humans created a way of explaining the world that works so well?
D	No, maths is only a tool. A hammer is not made better because it is more beautiful; the best hammer is one that works the best. The same is true for maths.

Do you think that mathematics is essential for living in the modern world?

A	Mathematical literacy is important for navigating the world we live in, but pure maths is valuable in and of itself.
B	Mathematical processes such as generalising, conjecturing, abstracting, symbolising, structuring, carrying out algorithms and justifying are essential for living in the modern world.
C	Yes! We only truly learn maths when we engage with the maths embedded in our own lives and environments.
D	No, but we need to prepare students who want to continue with maths-heavy professions such as science, engineering and finance.

Do you think that some people are naturally more talented at mathematics than others?

A	Yes, some people are just better at numbers and logical thinking than others.
B	Mathematical thinking skills can be learnt through hard work.
C	No, maths is a human creation, so all humans should be able to do it if taught properly.
D	There are many ways of coming to the right answer, everyone will probably be able to find a way that works for them.

If two students come up with different ways of arriving at the same answer, how do you decide which is the better answer?

A	The person who has the most logical answer has the best answer.
B	The person who has been most successful at generalising, and can give the best justification for why this would work in general, has the best answer.
C	The answer that displays the most critical thinking is the best answer.
D	If they both work to solve the problem, they are both equally valuable.

If you had to introduce a group of high performing students to the idea of non-Euclidean geometries, how would you introduce it?

A	I would begin by outlining the basic facts that apply to the new geometry.
B	I would point out that the same types of mathematical processes are taking place, but we are simply changing the starting axioms.
C	I would stress that these are two equally valuable ways of making sense of the world.
D	I would point out that a new geometry was created because new types of problems cropped up and had to be solved.

Do you think that mathematical truths are universally and objectively true?

A	Yes, because the best mathematical proofs are based on pure deductive reasoning.
B	Correctly engaging with mathematical processes is more important than knowing any given mathematical content.
C	No, we could have been using a base 12 counting system rather than our base 10 system. The maths we currently use is constructed by us, and therefore not infallible.
D	If maths ever stopped working in the real world, it would become false.

RESULTS

If you got mostly As…

You are most likely a Realist with regard to mathematics, and endorse the so-called Standard View. You probably believe that even if humans had never come into being, mathematical objects or relationships would nonetheless still have existed. When teaching, you probably focus on imparting mathematical facts and axioms to your students, and most likely view Mathematics as the objective observation of these entities. You also tend to lay emphasis on the skills necessary to carry out deductive reasoning, since through the use of deductive knowledge one can work carefully and systematically from axioms to conclusions that have mathematical certainty.

If you got mostly Bs…

You are most likely someone who revels in mathematical Practices and Processes. You probably focus a lot of your teaching on getting students to generalise, conjecture, abstract, symbolise and justify. You want your students to be autonomous inquirers and knowers in mathematics. These processes sometimes take preference over the specific mathematical content you are meant to be teaching. In order to show your students why these processes are useful, you probably hear yourself telling them that they are being equipped with problem solving methods which will help them to solve any problem that might come their way.

If you got mostly Cs…

You are most likely a Social Constructivist with regard to mathematics. This means that you view mathematical objects, relations and problems as man-made artefacts, and therefore see mathematics as a human activity which is culture-bound and value-laden. You probably agree that students come to the classroom with different mathematical conceptions and assumptions which need to be articulated, challenged and juxtaposed with other perspectives in order to allow the development of critical mathematical thinking and the understanding of the social institution of mathematics.

If you got mostly Ds…

You are most likely an Instrumentalist when it comes to mathematics. You probably believe that mathematics came about to solve problems in the physical world. You tend to value successful application of mathematics to the real world over mathematical certainty and rigour. You are inclined to see 'pure' mathematical skills, procedures, facts and knowledge as the 'dry bones' of mathematics, which are simply tools to be mastered. Applied mathematics, on the other hand, you see as the vital, living part of mathematics which can be used to solve problems related to science, engineering and finance.

WHAT DOES THIS ALL MEAN?

No single philosophy of mathematics is right or wrong. However, the particular philosophy of mathematics that we endorse nonetheless affects how we teach, what we teach and how we assess our students. To illustrate what I mean, let us consider a teacher who is trying to get students to appreciate that, when we look at the equation of a straight line function in standard form, i.e. $y = mx + c$, the c represents a vertical shift.

If a teacher had an Instrumentalist philosophy of mathematics, she might consider a real-life problem, and thus be of the opinion that the only thing that students need to know about c is that it is the y-intercept of the graph. If the teacher was a Realist with regard to mathematics, she might try to get students to appreciate that c represents the vertical shift through deductive and algebraic reasoning. This might be done, for example, by manipulating the straight line equation so that it can be seen that for any given x-value the addition of a positive constant would necessarily make the y-value bigger, i.e. would shift the graph vertically. In a test she would expect her students to be able to see at a glance, from the equation, what the vertical shift of a graph was. From the Realist view, pupils who merely understand that c is the y-intercept have missed out on important conceptual understanding necessary to appreciating the straight line equation.

If the teacher's philosophy centred on mathematical Practices and Processes she might assign an investigation so that students could explore the effect of c on the position of the graph. From the investigation she would encourage students to form a conjecture, come up with a generalisation, and justify why they thought this would always be the case. She would then expect students to be able to use these generalisations in a new context, for example seeing that the q value in the equation of a parabola of the form $y = x^2 + q$ would function in the same way.

If the teacher had a Social Constructivist view of mathematics she might investigate the misconceptions about straight lines that students come into her class with (e.g. that c is just the y-intercept of the graph). She would see these misconceptions not as something to be frustrated by, but as an opportunity to enhance learning. She might then try to create some form of cognitive conflict between the misconception and other pieces of the student's knowledge, for example by comparing the equations $y = 2x + 3$ and $y = 2^x + 3$ where the $+3$ represents the y-intercept of the former but not of the latter. Students might then be encouraged to discuss, reflect on and negotiate the given scenario with their peers.

WHY SHOULD WE CARE?

Each philosophy of mathematics has its own pros and cons. The Instrumentalist view might be too sparse, the Standard View might be too abstract, the Practices and Processes view might focus too little on specific content, while the Social Constructivist view might be time consuming.

However, I want to suggest that we should take note of what our philosophy of mathematics is for another reason. Since our own philosophy influences the way we teach, what we consider important, and how we assess, this is likely to have an influence on which students will be engaged and thus on which students are likely to succeed. From the Standard View, for instance, it is often believed that some pupils have built-in raw brilliance and mathematical ability, while others do not. If students feel that they lack this innate talent for mathematics it is unlikely that they will feel motivated to study mathematics. Implementing a pedagogy based on the Standard View runs the risk of excluding these students. Even the Instrumentalist view doesn't escape such worries, because if students do not want to pursue careers where real-life application of mathematics would be applicable, they might come to view mathematics as being irrelevant to their lives. The Practices and Processes view might unintentionally exclude those students who see mathematical success in terms of content mastery. Finally, the Social Constructivist view, with its focus on discussion and reflection, might perhaps exclude those who are not studying in their mother-tongue or who are not confident sharing their mathematical experiences.

TAKE-HOME POINT

Every Mathematics teacher will have a philosophy of mathematics that most strongly resonates with them, and which in turn influences how they teach and what aspects of mathematics they view as being important and relevant. This article is not meant to advocate for any particular philosophy of mathematics, nor does it suggest that such philosophies are mutually exclusive. However, given the potential influence that our philosophy of mathematics can have on our classroom practice, the aim of the article is to encourage teachers to reflect on their own practice in relation to their philosophy of mathematics. Are there students in your class who are potentially being excluded from participating because their view of mathematics is different to your own? The six question quiz at the start of this article is a useful way of sounding out your own views about mathematics and its teaching. It could also be a useful way of gaining insight into how your students view mathematics, which in turn may open up valuable classroom discussion.

REFERENCES

Department of Basic Education. (2011). *Curriculum and Assessment Policy Statement (CAPS), Mathematics: Further Education and Training Phase Grades 10-12*. Pretoria: Government Printing Works.

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