On the Areas of Triangles Bounded by Non-Parallel Sides and Diagonals of a Trapezium

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INTRODUCTION

Consider a trapezium ACDB (not necessarily isosceles) with diagonals AD and BC intersecting at E, as illustrated in Figure 1. Since ΔACD and ΔBCD have equal bases and equal heights, their areas are the same. Now, since ΔCDE is a common portion of these two triangles, it follows that the area of ΔACE is equal to the area of ΔBDE.

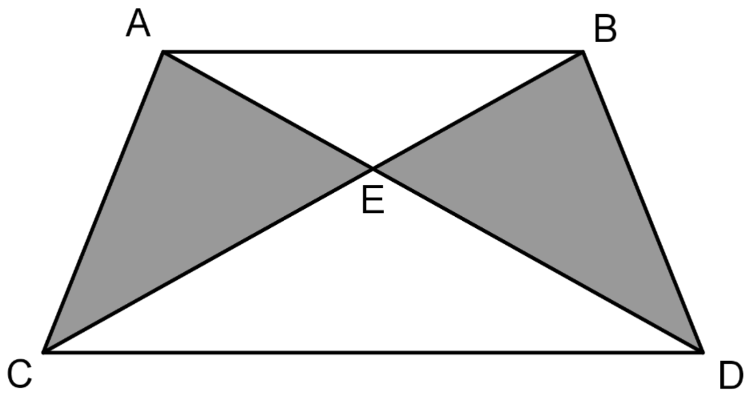

FIGURE 1: Area of ΔACE = Area of ΔBDE

This property is often not explicitly stated in school mathematics textbooks, but it turns out to be a very useful result for solving a number of geometry problems, as illustrated below.

EXAMPLE 1

In Figure 2, ABCD and BEFG are squares. If the area of BEFG is a cm^2, what is the area of ΔDEG in terms of a?

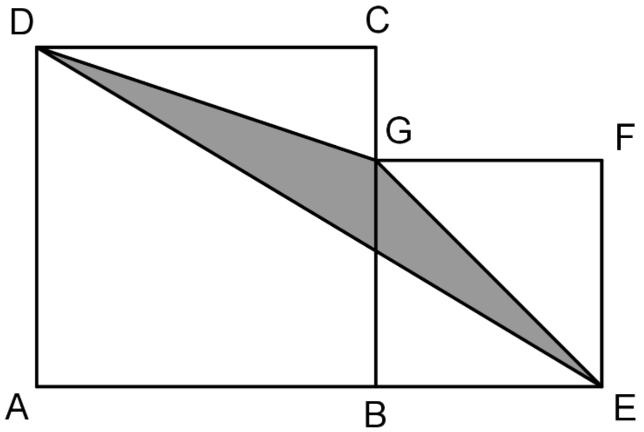

FIGURE 2: What is the area of ΔDGE?

Solution:

Draw diagonal BD as shown in Figure 3.

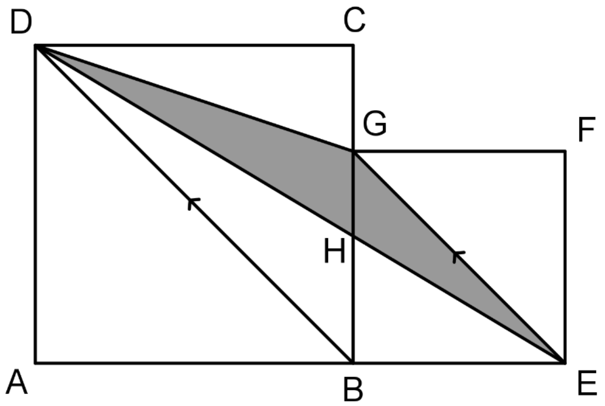

FIGURE 3: Draw diagonal BD to form trapezium DBEG

Since DB//GE, this means that DBEG is a trapezium. Using the property discussed earlier it follows that area of ΔDHG = area of ΔHBE. From this we can conclude that:

$$\text{area of } \Delta\text{DEG} = \text{area of } \Delta\text{BEG} = \frac{a}{2}\ \text{cm}^2$$

EXAMPLE 2

In Figure 4, ABCD is a trapezium with diagonals AC and BD intersecting at E. The areas of ΔCDE and ΔABE are m and n respectively. Find the area of ABCD in terms of m and n.

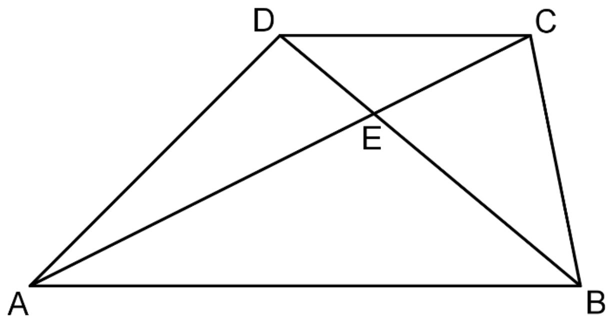

FIGURE 4: ABCD is a trapezium

Solution:

Let the area of ΔCBE equal s. Since ABCD is a trapezium, the area of ΔADE is also equal to s. Looking at ratios we now see that $\frac{\text{area }\Delta AED}{\text{area }\Delta ECD} = \frac{AE}{EC} = \frac{\text{area }\Delta AEB}{\text{area }\Delta ECB}$. We thus have $\frac{s}{m} = \frac{n}{s}$ from which we can conclude that $s = \sqrt{mn}$. Hence, the area of ABCD $= m + n + 2\sqrt{mn}$.

FINAL REMARKS

In this article we have described a property about the areas of triangles bounded by the diagonals and the non-parallel sides of a trapezium. We hope this property can be found useful for reference by school teachers and students who are studying elementary plane geometry.