Using Real World Contexts to Explore Fundamental Mathematical Ideas

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Introduction

There is a solid rationale for infusing the middle level and high school Mathematics curriculum with robust mathematical ideas derived from real world contexts (see e.g. NCTM 2000; Koedinger & Nathan, 2004). Using this rationale as a base, this article poses non-trivial mathematical problems that originated in a context outside the classroom, and analyses the fundamental mathematical content of the problems. The core concepts relate to ratio and proportional reasoning as well as the interpretation of graphical information and possible misconceptions related thereto.

The purpose of this article is twofold. The first is to share a short module of work that arose from real life experiences of the author. The module takes the form of problems and activities, and could be suitable for either middle level or secondary students. The second objective is to demonstrate that serious and significant mathematical ideas can appear in seemingly mundane activities.

Mathematics through a story

This paper presents a story that illuminates for the reader how everyday encounters in the non-school world contain legitimate opportunities for mathematical reasoning. By no means is a claim being made that this particular context possesses the same level of interest for everyone else that it does for the author. However, a claim *is* being made that just as this example motivates the author, different contextual scenarios can be found to inspire a diverse population of students.

The story: Planning a hiking trip

When planning a hiking trip, many features of the route are important to note. For example, a hiker might care to know how long the trail is, whether it can be hiked as a loop, or only as an "out and back", how rough the terrain is, whether water is available along the route, and so on. Beyond those concerns, another important consideration is the steepness of the terrain. Is the climb long but gradual? Is it short but steep? Or is it some combination of the two? This kind of information can sometimes be found in guidebook descriptions, but can also be gleaned from published elevation profiles (graphs that depict elevation along one axis and distance from the starting point on the other axis).

Elevation profiles utilize a map-making technique that gives the viewer a range of information, including the relative steepness of the terrain, either localized to a particular portion or averaged over the entire distance of the route. In addition to the numerous fundamental mathematical ideas involved, such as slope, ratio, scale, and rates of change, another fascinating aspect of elevation profiles is how easily the viewer can be deceived by the visual representation. This is especially true when different regions and trails of varying lengths are presented using different scales. These ideas form the basis of this article.

COMPARING ELEVATION PROFILES

The following task is based on a number of elevation profiles and offers a series of questions in which students need to use mathematical reasoning to generate solutions. The purpose is to highlight the authentic opportunities for reasoning and sense making through the use of real life graphical data.

Three elevation profiles are depicted below. Each represents a real hike from a favourite park in the author's home state. The inspiration for this paper, and these activities, is that the graphs did not match the author's recollection of the steepness of each of the trails.

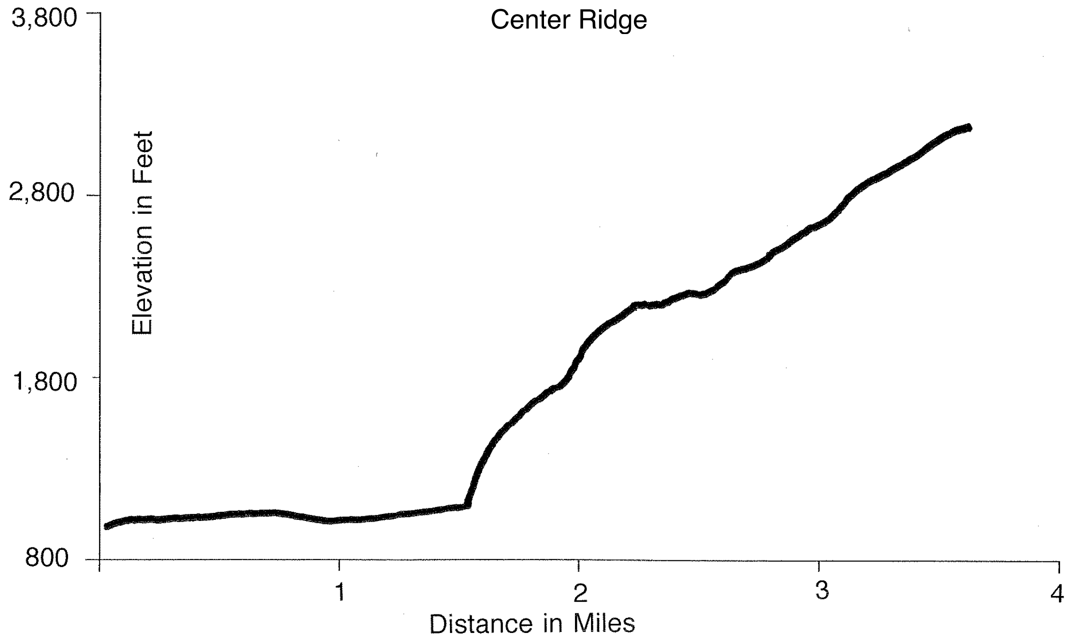

FIGURE 1: Elevation profile of Center Ridge

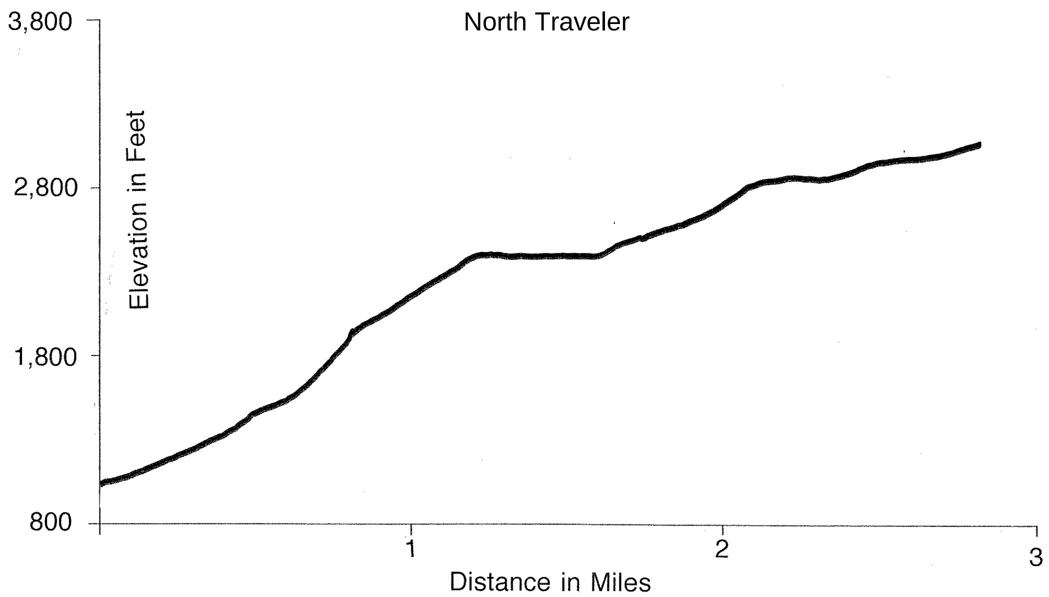

FIGURE 2: Elevation profile of North Traveler

FIGURE 3: Elevation profile of South Branch

TASK 1

Study the three elevation profiles and answer the following questions:

1. On the basis of a quick visual scan, which hike appears to be steepest?
2. What information do you need to answer the above more thoroughly? Is that information provided in the diagrams?
3. Using the data provided in the elevation profiles:
   (a) Devise a technique for determining which hike is the steepest.
   (b) Explain on what basis you decided which hike was steepest.
   (c) Does this answer match your initial visual assessment?
4. Is it possible for a given hike to have the steepest section of trail, yet not be the steepest trail overall? What might a graph of this example look like?
5. Certain sections of each trail appear to be particularly steep. Examine and compare each of the following:
   - Center Ridge Trail from mile 1.5 to the finish
   - North Traveler from mile 0 to the end of the first pitch (approximately mile 1.2)
   - South Branch from mile 0 to the first summit, at mile 2.

   How does the steepness of each of these sections appear visually? Now, using your best estimates and approximations, calculate the vertical feet gained per each mile of distance. Discuss how these results compare to the visual impression.

EXTENDING THE IDEA

The following two tasks further develop the idea of slope or gradient.

TASK 2

The following four graphs each appear, at an initial glance, to have a different 'slope' or 'steepness'. Study the four graphs and answer the questions that follow.

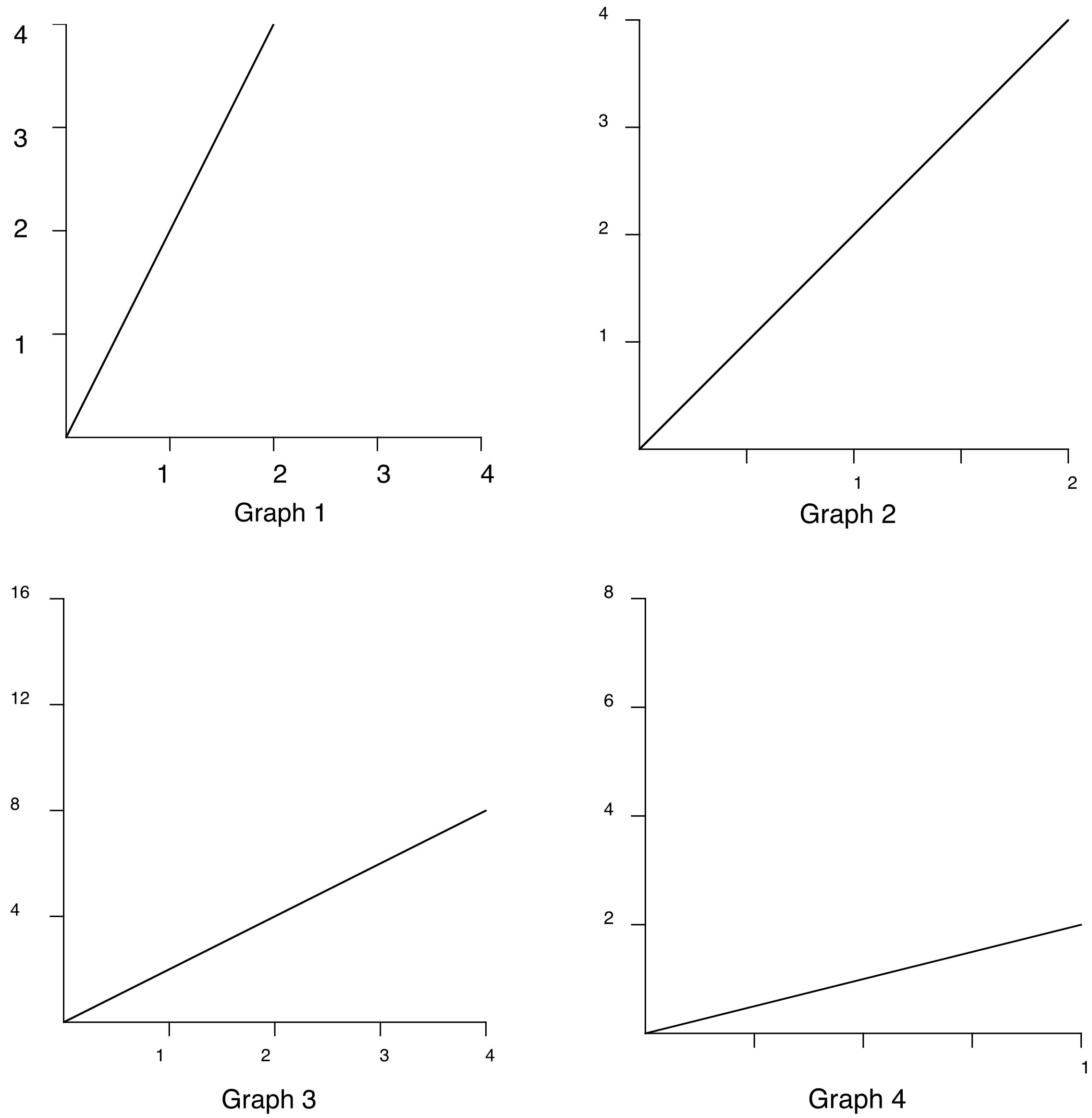

FIGURE 4: Four graphs indicating slope

1. Use the scales provided on the axes to determine the slope of each graph.
2. Write an explanation describing the manner in which changing the scale on either axis 'changes' the slope of a line.
3. Building on your above answer, explain how graphical information might be manipulated to purposefully distort the information. Describe a context within the social, cultural, political or economic world where this might happen.

TASK 3

Study the four graphs shown below. You will notice that Graph A has a scale for the both axes whereas Graphs B, C and D have missing scales.

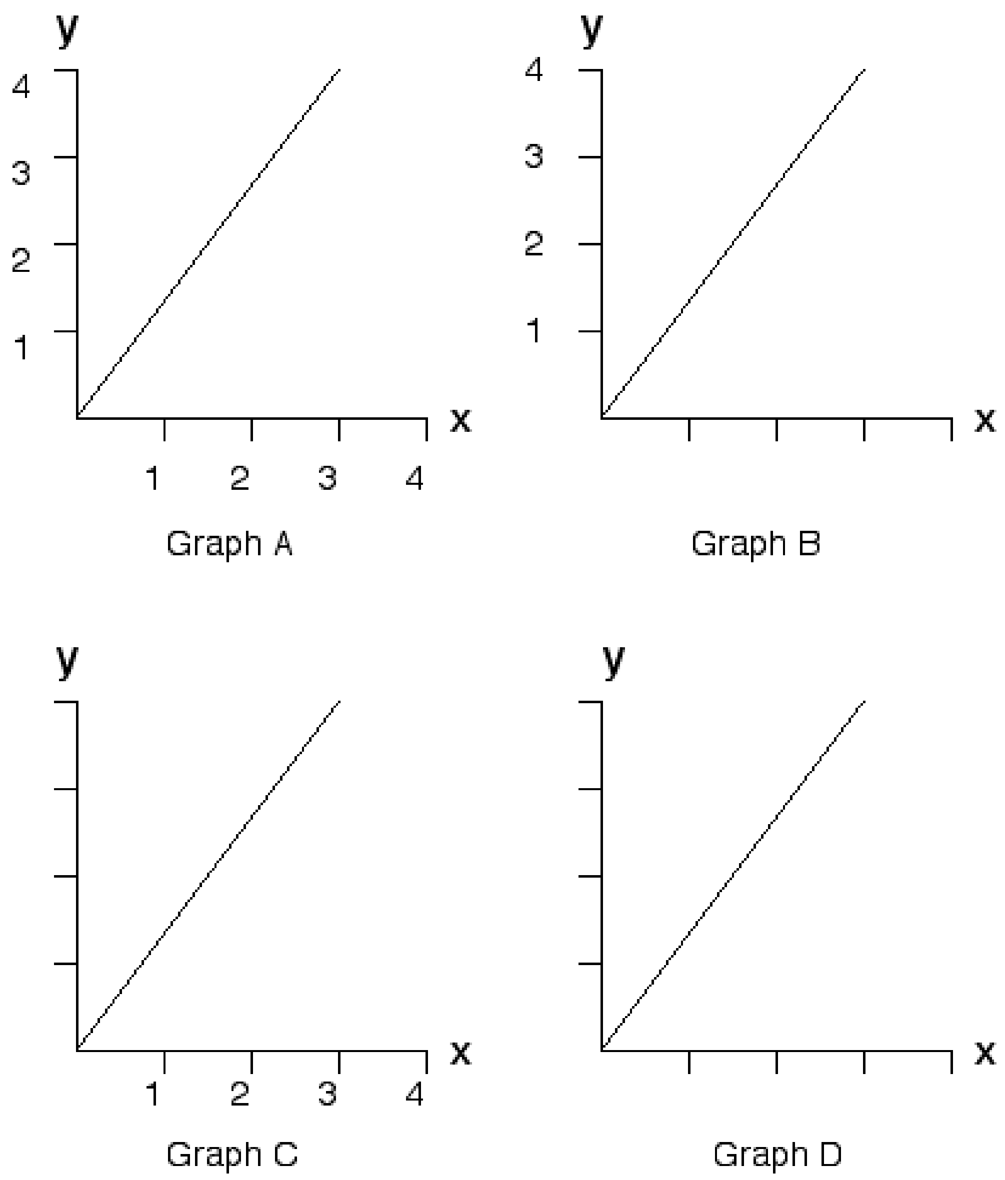

FIGURE 5: Four graphs, some with missing scales

Your task is the following:

- On Graph B, choose a scale for the *x*-axis such that Graph B has a greater slope than Graph A
- On Graph C, choose a scale for the *y*-axis such that Graph C has a greater slope than Graph A
- On Graph D, choose a scale for both axes such that Graph D has a greater slope than Graph A

CONCLUDING COMMENTS

The scenario described in this paper derives from a context of particular interest to the author. The accompanying mathematical ideas and tasks are by no means intended to be exhaustive. Their purpose is to serve as an exemplar of how educators can link school mathematics and real world mathematics, using each to strengthen the other. While on first inspection it may seem that the major message of this article is that graphs can be deceptive, or that it's possible to mislead with visual presentations, the underlying mathematical idea is something much more subtle, namely the concept of gradient or slope. Slope is a function of the scales of both axes. It is a rate of change, not simply an impression we form based on the apparent angle of a line segment.

The aim of putting together these various activities was to build curricular and instructional approaches that enhance students' grasp of fundamentally important, often abstract, mathematical concepts by linking those concepts to a meaningful, concrete and robust context. The field of mathematics education repeatedly calls for this linking of concrete and abstract, with the former serving as a foundation for the latter. Given the emphasis on student engagement as a critical factor of success, finding ways to motivate students in mathematical inquiry becomes increasingly important. The challenge for educators is to find contexts that inspire students.

REFERENCES

Koedinger, K. R., & Nathan, M. J. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *Journal of the Learning Sciences 13*(2), 129-164.

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