Adding Tops and Bottom of Fractions

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I would like to share with fellow mathematics educators and teachers my experience with some primary school children on addition of fractions. Some months ago I worked with standard 6 and 7 children (about 10-13 years old) in Malawi. One of the topics was addition of fractions. I observed that the most common error that the students made was to add the numerators and add the denominators. For example to add $\frac{2}{3}$ and $\frac{1}{4}$ they would write $\frac{2}{3} + \frac{1}{4} = \frac{3}{7}$.

During discussions with some of the students in groups, some said they add the tops and the bottoms because their teachers do it sometimes. I was sure they were mistaken so I asked them to show me an example of what the teacher did. One boy, Ndida, who had been rather quiet the whole time, raised his hand and said that the teacher had done it on his test script. He illustrated that the teacher wrote, for example, 23/40 + 45/60 = 68/100. I looked at Ndida's illustration and tried to find a way to explain to the children why the teacher could do this when adding up test marks but that as learners they should not do this.

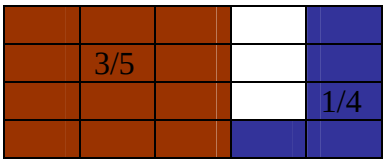
What Ndida's teacher did is quite common and many of us teachers do it all the time. Another example of this kind of addition is the school report at the end of term. A typical school report for a primary school child in Malawi would have a section of marks that looks like the following:

Mathematics	85/100
English	60/100
Science	62/100
Geography	71/100
History	57/100
Total	335/500

Again here the teacher would have added the top numbers (numerators) together and the bottom numbers (denominators) together. So why is it that teachers can do it sometimes but the children are not allowed to do it at all?

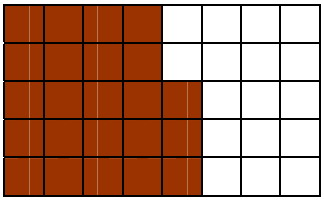
What does it mean to add fractions?

Let's take a simple example of $\frac{3}{5} + \frac{1}{4}$. This means we want to add three fifths of one whole to one quarter of the **same or identical** whole. The lowest common denominator is 20 so it is easiest to use a 'whole of 20' like the grid below. Adding the two fractions means adding $\frac{3}{5}$ of the 20 (which is 12) to $\frac{1}{4}$ of the same 20 (which is 5). So we get (12+5) out of 20 giving us the answer of 17/20.



What are teachers really doing when they add tops and bottoms?
When Ndida's teacher did something like, 23/40 + 45/60 = 68/100, what was she actually doing? And why does it work?

The total marks for the test was 100, therefore the whole in this case is 100. She had divided the test into two sections and so she split the whole into 40 and 60. Ndida got 23 out of 40 in section A and 45 out of 60 in section B and altogether he got 68 out of 100.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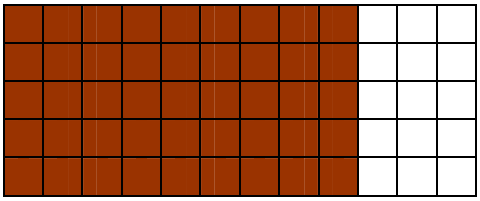
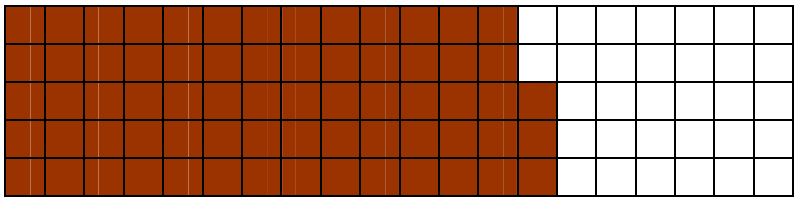


put together we get



23/40 and 45/60 are **not added as fractions** because they are **not parts of the same whole**. If we want to write these as fractions then we have to use the whole of 100 and so we will have 23/100 and 45/100. In terms of fractions, Ndida got twenty-three hundredths in section A and forty-five hundredths in section B, and his total was sixty-eight hundredths.

The same applies to school reports. The whole in the example given is 500, and it has been split into 5 sections of 100 each. The addition that takes place is not that of fractions. Again here in terms of fractions we would have

$$\frac{85}{500} + \frac{60}{500} + \frac{62}{500} + \frac{71}{500} + \frac{57}{500} + \frac{57}{500} = \frac{335}{500}.$$

As teachers, we need to be aware of both what we are doing and what students might think we are doing.